THE SEMICIRCLE ON A SECTOR:

J. Doody

In two Books.

CONTAINING
The deſcription of a general and portable Inſtrument; whereby moſt Problems (reducible to Inſtrumental Practice) in *Aſtronomy*, *Trigonometry*, *Arithmetick*, *Geometry*, *Geography*, *Topography*, *Navigation*, *Dyalling*, &c. are ſpeedily and exactly reſolved.

By *J. T.*

LONDON,
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MVSEVM
BRITAN
NICVM

To the Reader.

ALL that is intended in this Treatiſe, is to acquaint thee with an Inſtrument, that is both portable and general, of no great price, eaſie carriage, yet of a ſpeedy and Accurate diſpatch in the moſt difficult Problems in Aſtronomy, &c. *The lines for the moſt part have been formerly publiſhed by Mr.* Gunter, *the famous Mr.* Foſter, *Mr.* White, &c. *The reduction of the* 28. *Caſes of Spherical Triangles unto* 11. *Problems, I firſt learned from the reverend Mr.* Palmers *Catholick Planiſphere. Many of the proportions in the Treatiſe of Dyalling are taken from (though firſt compared with the Globe) my worthy Friend (to whom I am indebted in all the Obligations of Civility, and without whoſe encouragement this had never adventured the publick Teſt) Mr.* John Collins. *The applying Mr.* Foſters *Line of Verſed Sines unto the Sector was firſt publiſhed by Mr.* John

Brown, *Mathematical Inſtrument-Maker, at the Sphere and Sun-Dial in the* Minories, London, Anno 1660. *who hath very much aſſiſted me, by making, adding unto, and giving me freely the peruſal of many Inſtruments, according to any directions for Improvement, that was propoſed to him. After this account, what hath been my part in this Work, I hazard to thy cenſure; and when I ſee others publiſh a more convenient, ſpeedy, accurate, and general Inſtrument, I aſſure them to have as low thoughts of this, as themſelves. But here is ſo large a Catalogue of* Errata's *as would ſtagger my confidence at thy pardoning, had they not been irrevocably committed before I received the leaſt notice of them. The Printer writing me word (after I had corrected ſo much as came to my ſight) that he could alter no Miſtakes until the whole Book was printed: By which means he enforced me to do pennance in his Sheets, for his own Crimes: Did not one groſs miſtake of his become my purgation,* viz. *in* lib. 2. *(through-*

(throughout Chap. 3.) where inſtead of the note of equality (marked thus =) he hath inſerted the Algebraick note of Subtraction, or Minoration (marked thus -)

Nor hath the Engraver come behinde the Compoſer, who ſo miſerably mangleth Fig. 13. that (at firſt ſight) it would endanger branding of a mans Brains to ſpell the meaning thereof, either in it ſelf, or in reference to the Book. All that I can help thee herein, is this; Whereas the Book mentions that Figure for an Eaſt Dyal, if you account it (as now cut) a Weſt Dyal, and alter the names of the hours, by putting Figures for the afternoon, in the place of thoſe there for the morning; you will then have a true Weſt Dyal of that Figure.

The correction of Punctations would be an endleſs task; for I finde ſome to be reſolved, ever ſince Valentine, *to recreate themſelves at* Spurn-point.

What other material miſtakes are in the Book (which ought to be corrected before reading thereof) you will finde mentioned in the Errata. *Farewel.*

March 29. 1667. J. T.

Errata.

PAge 4. line 12. *ſigns*, r. *ſines*. p. 8. l. 5. *ſeconds*, r. *ſecants*. p. 12. l. 9. *all*, r. *allone*. p. 13. l. 9, and 10. *ſec*. r. *min*. p. 14. l. 17. *ſec*. r. *min*. p. 22. l. 14. *any*, r. *what*. p. 23. l. 3. *exact*, r. *erect*. p. 24. l. *ult*. adde *lib*. 2. p. 25. l. 2. *ſigns*, r. *ſines*. l. 5. *ſign*. r. *ſine*. p. 35. l. 3. *a mark*, r. *an ark*. p. 37. l. 22. 20. r. 22. and 42. r. 20. p. 44. l. 5. *At*, r. *As*. p. 49. l. 1. *divided by*, r. *dividing*. p. 62. l. 15. dele, *a*. In lib. 1. chap. 9. the pages are falſe numbred. But in chap. 9. p. 62. l. 11. *next*, r. *exact*. l. 15. *gauger*, r. *gauge*. p. 71. l. 24. *the*, r. *what*. p. 73. l. 5. *whereas*, r. *where I*. p. 80. l. 13. *wherein*, r. *whereof*. p. 86. l. 22. *pont*, r. *point*. p. 91. l. 1. *the co-tangent*, r. *half the co-tangent*. l. 19. *L*. r. *P*. p. 92. l. 21. *PZ*. r. *PS*. p. 93. l. 12. NSP, r. NSZ. p. 94. l. 6. *angle*, r. *ark*. throughout page 96. *Fig*. 3. r. *Fig*. 7. and *Fig*. 4. r. *Fig*. 8. p. 98. l. 13. *ſerve*, r. *ſcrue*. p. 99. l. 12. *places*, r. *plates*. l. 13. *proportion*, r. *perforation*. p. 100. l. 9. *ſerve*, r. *ſcrue*. p. 101. l. 4. *ſerve*, r. *ſcrue*. l. 21. *ſerve*, r. *ſcrue*. p. 111. l. 20. *lay in*, r. *laying*. p. 117. l. 14. *of*, r. *and*. p. 128. l. 3. *Fig*. 12. r. *Fig*. 13. l. 22. *ED*. r. *EC*.

In Fig. 12. *C*. r. *A*. at the end of the line *G*.

To the Right Honorable,

The Lord SHERARD,

Baron of *Letrim*.

My Lord,

SInce the trifling Treatiſe of an Almanack hath uſurped a cuſtom to pinnion ſome Honourable Name to the Patronage of the Authors Follies; had we not certain evidence from the uncertainty of their Predictions, that their Brains (like their great Oracles the Planets) are often wandring; it might be deemed a Crime, beyond the benefit of the Clergy, to prefix before any Book, a dedication to a Noble Perſon: Or when I read the unreaſonableneſs of others in thoſe Addreſſes, imploring their Patrons to be their *Dii Tutelares*, and proſtrate their reputes to the unmannerly mangling of every Cenſuriſt, under the notion of protecting (that is adopting) the Authors Ignorance, or negligence; its enough to tempt the whole world to turn Democritians, and hazard their ſpleens in laughing at ſuch mens madneſs. My preſent deſign is only to give your Lordſhip my obſervance of your Commands about the Deſcription and Improvement of the Sector; and wherein I have erred (through miſtake, or defect) I deſpair not, but from your Honor I ſhall meet with a pardon of Courſe to be granted unto

Your Lordſhips moſt

humble Servant

J. T.

THE SEMICIRCLE ON A SECTOR.

Lib. I.

Chap. I.

A Deſcription of the Inſtrument, with the ſeveral Lines inſcribed thereon.

THe Inſtrument conſiſts of three Rulers, or Pieces; two whereof are joyned together by a Rivet that may open and ſhut to any Angle, in faſhion of the Sector; or to uſe a courſer compariſon, after the manner of Compaſſes. The third piece is looſe, or ſeparable from them, to be put into the Tenons at the end of the inward ledge of the joyned pieces, and thereby conſtituting an æquilateral triangle.

On theſe Rulers (after this manner put together) we take notice (for diſtinction ſake) of the ſides, ledges, ends, and pieces.

The ſides are thus differenced, one we call the quadrantal, the other the proportional, or ſector ſide.

The ledges are diſtinguiſhed by naming one the inward, the other the outward ledge.

The ends are known in terming one the head (*viz.* where the two pieces are riveted together) the other the end.

The pieces are diſcovered by ſtyling one the fixed piece, *viz.* that which hath the rivet upon it; the ſecond the movable piece, which turns upon that rivet, and the laſt the looſe piece, to be put into the tenons, as before expreſſed.

The quadrantal ſide of the joyned pieces is eaſily diſcerned, having the names of the Moneths ſtamped on the movable piece, and Par. Scale on the fixed piece. The quadrantal ſide of the looſe piece is known by the degrees on the inward and outward limb.

Theſe directions are ſufficient to inſtruct you how to put the Inſtrument together.

Imagining the Inſtrument thus put together, the lines upon the quadrantal ſide are theſe.

Firſt on the fixed piece, next the outward limb, is a line of 12 equal parts; and each of thoſe parts divided into 30 degrees, marked from the end towards the head with ♈ ♉ ♊, *&c.* repreſenting the 12 Signs of the Zodiack; the uſe of this line, with the help of thoſe under it, was intended to find the hour of the night by the Moon.

The next line to this is a line of twice 12, or 24 equal parts, each diviſion whereof cuts every 15th degree of the former line: and therefore if the figures were ſet to every 15th degree on this ſide the former line, this ſecond line would be uſeleſs, and the former perform its office more diſtinctly. This line was intended an aſſiſtant for finding the hour by the Moon; but is very ready to find the hour by any of the fixed Stars.

The third line is a line of 29½ equal parts, ſerving for the dayes of the Moons age; in order to find the hour of the night by the Moon. But the operation is ſo tedious, and far from exactneſs, that I have no kindneſs for it; and ſhould place ſome other lines in the room of this and the former, did I not reſolve to impoſe upon no mans phantaſie.

The fourth line is a line of altitudes for a particular latitude, noted at the end, Par. Scale, *&c.* This helpeth to find the hour and azimuth of the Sun, or any fixed Star very exactly.

The fifth line is a line of natural ſines; at the beginning whereof there is a pin, or elſe an hole to put a pin into, whereon to hang the thread and plummet for taking of altitudes. To this line of ſines may be joyned a line of tangents to 45 degr. The uſe of the ſines alone, is to work proportions in ſigns. The uſe of the ſines with the tangent line may be for any proportion in trigonometry; but that I leave to liberty.

The ſixth line, and laſt on this ſide the fixed piece is a line of verſed ſines, numbred from the centre at the head to 180 at the end.

On the quadrantal ſide of the movable piece, the firſt line next the inward edge, is a line of verſed ſines anſwering to that on the fixed piece. The uſe of theſe verſed ſines is various at pleaſure.

The ſecond line from the inward edge, is a line of hours and Azimuths ſerving to find the hour by the Sun or Stars, or the Azimuth of the Sun, or any fixed Star from the South.

The third and fourth lines are lines of Moneths, marked with the reſpective names; and each Moneth divided into ſo many parts as it contains dayes.

The fifth line is a line of ſigns marked ♈, ♉ ♊, *&c.* each ſign being divided into 30 degr. and proceeding from ♈, or *Aries* (which anſwers to the tenth of *March*) in the ſame order as the Moneths. The uſe of this line, with help of the Moneths, is to find the Suns place in the Zodiack.

The ſixth line is a line of the Suns right aſcenſion, commonly noted by hours from 00 to 24. but better if divided by degrees or ſines from 00 to 360, and both wayes proceeding backward and forward as the ſigns of the Zodiack, or dayes of the Moneth.

Laſtly the outward edge or limb of the movable and looſe piece both, is graduated unto 180 degrees, or two quadrants; whoſe centre is the pin, or pin-hole before mentioned, at the beginning of the ſines on this ſide the fixed piece. The perpendicular is at 0/60 upon the looſe piece; from whence reckoning along the outward edge of the looſe piece, till it interſects the produced line of ſines at the end of the fixed piece, you have 90 degrees. Or counting from 0|60 on the looſe piece, and continuing it along the degrees of the outward limb of the movable piece, until they interſect the produced line of ſines on the fixed piece at the head, you have again 90 degrees, which compleat the Semicircle. The uſe of this line is for taking of altitudes, counting upon the former 90. degr. when you hold the head of the fixed piece toward the Sun; and numbring upon this latter, when you hold (which is beſt because the degrees are largeſt) the end of the fixed piece toward the Sun. There are other wayes of numbring theſe degrees for finding of the Azimuth, *&c.* which ſhall be mentioned in their proper places.

On the quadrantal ſide of the looſe piece, the inward edge or limb is graduated unto 60 degrees (or twice 30, which you pleaſe) whoſe centre is a pin at the head. The uſe of this is to find the altitude of the Sun, or any Star, without thread and plummet; or to perform ſome uſes of the Croſs-ſtaff. This is for large rules or inſtruments, and therefore not illuſtrated here.

In the empty ſpaces upon the quadrantal ſide may be engraven the names of ſome fixed Stars, with their right aſcenſions and declinations.

On the proportional ſide the lines iſſuing from the centre are the ſame upon the fixed and movable piece; but happily tranſplaced (thanks to the firſt contriver) after this manner: The line that lies next the inward edge on the fixed piece hath his fellow or correſpondent line toward the outward edge on the movable piece; by which means theſe lines all meeting at the centre, ſtand all at the ſame angle, and give you the freedom from a great deal of trouble, in working proportions by ſines and tangents, or laying down any ſine or tangent to any Radius given, *&c.*

The lines iſſuing from the centre toward the outward edge of the movable piece, whoſe fellow is next the inward edge of the fixed piece, is a line of natural ſines on the outward ſide, marked at the end S, and on the inward ſide a line of lines or equal parts, noted at the end *L*; the middle line ſerving for both of them.

The lines iſſuing from the centre next the inward edge of the movable piece, whoſe fellows are toward the outward edge of the fixed piece, are lines of natural tangents, which on the outward ſide of the line is divided to 45 the Radius; and on the inward ſide of the lines (the middle line ſerving for both) at a quarter of the former Radius from the centre is another Radius noted 45 at the beginning, and continued to the tangent 75.

Theſe lines are noted at the end *T*. The uſe of theſe you will find *Chap.* 3, 4, 5.

Betwixt the lines of ſines and tangents, both upon the fixed and movable pieces, is placed a line of ſeconds, continued unto 60, and marked at the end *Se*.

Next to the outward edge on the fixed and movable piece (which is beſt diſcerned when thoſe pieces are opened to the full length) is a line of *Meridians* divided to 85; whoſe uſe is for Navigation, in deſcribing Maps or Charts, *&c.*

In the vacant ſpaces you may have a line of chords, ſines, and tangents, to any Radius the ſpace will bear; and what other any one thinks beſt of, as a line of latitudes and hours, *&c.*

On the proportional ſide of the looſe piece are lines for meaſuring all manner of ſolids, as Timber, Stone, *&c.* likewiſe for gaging of Veſſels either in Wine or Ale meaſure.

On the outward ledge of the movable and fixed piece, both (which in uſe muſt be ſtretched out to the full length) is a line of artificial numbers, ſines, tangents, and verſed ſines. The firſt marked *N*, the ſecond *S*, the third *T*, the fourth *VS*.

On the inward ledge of the movable piece is a line of 12 inches divided into halfs, quarters, half-quarters. Next to that is a prick'd line, whoſe uſe is for computing of weight and carriages. Laſtly a line of foot meaſure, or a foot divided into ten parts; and each of thoſe ſubdivided into ten or twenty more.

On the inward ledge of the looſe piece you may have a line of circumference, diameter, ſquare equal, and ſquare inſcribed.

There will ſtill be requiſite ſights, a thread and plummet. And if any go to the price of a ſliding Index to find the ſhadow from the plains perpendicular, in order to taking a plains declination, and have a ſtaff and a ball-ſocket, the Inſtrument is compleated with its furniture. Proceed we now to the uſes.

Onely note by the way that Mr. *Brown* hath (for conveniency of carrying a pair of Compaſſes, Pen, Ink, and Pencil) contrived the fixed piece and movable both to be hollow, and then the pieces that cover thoſe hollows do, one ſupply the place of the looſe piece for taking altitudes; the other (being a ſliding rule) for meaſuring ſolids, and gaging Veſſels without Compaſſes.

CHAP. II.

Some uſes of the quadrantal ſide of the Inſtrument.

PROBL. I.

To find the altitude of the Sun or any Star.

HAng the thread and plummet upon the pin at the beginning of the line of ſines on the fixed piece, and (having two ſights in two holes parallel to that line) raiſe the end of the fixed piece, toward the Sun until the rayes paſs through the ſights (but when the Sun is in a cloud, or you take the altitude of any Star, look along the outward ledge of the fixed piece, until it be even with the middle of the Sun or Star) then on the limb the thread cuts the degree of altitude, if you reckon from 0|60 on the looſe piece toward the head of the movable piece.

PROBL. 2.

The day of the Moneth given, to find the Suns place, declination, aſcenſional difference, or time of riſing and ſetting, with his right aſcenſion.

The thread laid to the day of the Moneth gives the Suns place in the line of ſigns, reckoning according to the order of the Moneths (*viz.* forward from *March* the 10th. to *June*, then backward to *December*, and forward again to *March* 10.) In the limb you have the Suns declination, reckoning from 60/0 on the movable piece towards the head for North; toward the end for South declination. Again, on the line of right aſcenſions, the thread ſhews the Suns right aſcenſion, in degrees, or hours, (according to the making of your line) counting from *Aries* toward the head, and ſo back again according to the courſe of the ſigns unto 24 hours, or 360 degrees. Laſtly on the line of hours you have the time of Sun riſing and ſetting, which turned into degrees (for the time from ſix) gives the aſcenſional difference. Ex. gr. in lat. 52. deg. 30 min. for which latitude I ſhall make all the examples.

The 22 day of *March* I lay the thread to the day in the Moneths, and find it cut in the Signs 12 deg. 20 min. for the Suns place, on the limb 4 deg. 43 min. for the Suns declination North. In the line of right aſcenſions it gives 46 min. of time, or 11 deg. 30 min. of the circle. Laſtly, on the line of hours it ſhews 28 min. before ſix for the Suns riſing; or which is all, 7 deg. for his aſcenſional difference.

PROBL. 3.

The declination of the Sun or any Star given to find their amplitude.

Take the declination from the ſcale of altitudes, with this diſtance ſetting one point of your Compaſſes at 90 on the line of Azimuth, apply the other point to the ſame line it gives the amplitude, counting from 90 Ex. gr. at 10 deg. declination, the amplitude is 16 deg. 30 min. at 20 deg. declination, the amplitude is 34 deg.

PROBL. 4.

The right aſcenſion of the Sun, with his aſcenſional difference, given to find the oblique aſcenſion.

In Northern declination, the difference betwixt the right aſcenſion and aſcenſional difference, is the oblique aſcenſion. In Southern declination take the ſumm of them for the oblique aſcenſion, Ex. gr. at 11 deg. 30 ſec. right aſcenſion, and 6 deg. 30 ſec. aſcenſional difference. In Northern declination the oblique aſcenſion will be 5 deg. in Southern 18 deg.

PROBL. 5.

The Suns altitude and declination, or the day of the Moneth given to find the hour.

Take the Suns altitude from the Scale of altitudes, and laying the thread to the declination in the limb (or which is all one, to the day in the Moneths) move one point of the compaſſes along the line of hours (on that ſide the thread next the end) until the other point juſt touch the thread; then the formet point ſhews the hour; but whether it be before or after noon, is leſt to your judgment to determine. Ex. gr. The 22 day of *March*, or 4 deg. 43 min. North declination, and 20 deg. altitude, the hour is either 47 minutes paſt 7 in the morning, or 13 minutes paſt 4 afternoon.

PROBL. 6.

The declination of the Sun, or day of the Moneth, and hour given to find the altitude.

Lay the thread to the day or declination, and take the leaſt diſtance from the hour to the thread, this applyed to the line of altitudes, gives the altitude required. Ex. gr. The 5 day of *April* or 16 deg. declination North, at 7 in the morning, or 5 afternoon, the altitude will be 17 deg. 10 ſec. and better.

PROBL. 7.

The declination and hour of the night, given to find the Suns depreſſion under the horizon.

Lay the thread to the declination on the limb; but counted the contrary way, *viz.* from 60/0 on the movable piece toward the head for Southern; and toward the end for Northern declination. This done take the neareſt diſtance from the hour to the thread, and applying it to the line of altitudes, you have the degrees of the Suns depreſſion. *Ex. gr.* at 5 deg. Northern declination, & 8 hours afternoon, the depreſſion is 13 deg. 30 min.

PROBL. 8.

The declination given to find the beginning and end of twilight, or day-break.

Lay the thread to the declination counted the contrary way, as in the laſt Problem, and take from your Scale of altitudes 18 deg. for twilight, and 13 deg. for day-break, or clear light; with this run one point of the Compaſſes along the line of houres (on that ſide next the end) until the other will juſt touch the thread, and then the former point gives the reſpective times required. Ex. gr. At 7 deg. North declination, day breaks 8 minutes before 4: but twilight is 3 houres 12 minutes in the morning, or 8 hours 52 minutes afternoon.

PROBL. 9.

The declination and altitude of the Sun or any Star given to find their Azimuth in Northern declination.

Lay the thread to the altitude numbred on the limb of the moveable piece from 60/0 toward the end (and when occaſion requires, continue your numbring forward upon the looſe piece) and take the declination from your line of altitude; with this diſtance run one point of your Compaſſes along the line of Azimuths (on that ſide the thread next the head) until the other juſt touch the thread, then the former point gives the Azimuth from South. Ex. gr. at 10 deg. declination North, and 30 deg. altitude, the Azimuth from South is 64 deg. 40 min.

PROBL. 10.

The Suns altitude given to find his Azimuth in the equator.

Lay the thread to the altitude in the limb, counted from 60/0 on the looſe piece toward the end, and on the line of Azimuths it cuts the Azimuth from South. Ex.gr. at 25 deg. altitude the Azimuth is 53 deg. At 30 deg. altitude the Azimuth is 41 deg. 30 min. *fere.*

PROBL. 11.

The declination and altitude of the Sun, or any Star given to find the Azimuth in Southern declination.

Lay the thread to the altitude numbred on the limb from 60/0 on the moveable piece toward the end, and take the declination from the Scale of altitudes; then carry one point of your Compaſſes on the line of Azimuths (on that ſide the thread next the end) until the other juſt touch the thread, which done, the former point gives the Azimuth from South. Ex. gr. at 15 deg. altitude and 6 deg. South declination the Azimuth is 58 deg. 30 min.

PROBL. 12.

The declination given to find the Suns altitude at Eaſt or Weſt in North declination, and by conſequent his depreſſion in South declination.

Take the declination given from the Scale of altitudes, and ſetting one point of your Compaſſes in 90 on the line of Azimuths, lay the thread to the other point (on that ſide

90 next the head) on the limb it cuts the altitude, counting from 60/0 on the moveable piece. Ex. gr. at 10 deg. declination the altitude is 12 deg. 40 min.

PROBL. 13.

The declination and Azimuth given to find the altitude of the Sun or any Star.

Take the declination from the Scale of altitudes; ſet one point of your Compaſſes in the Azimuth given, then in North declination turn the other point toward the head, in South toward the end; and thereto laying the thread, on the limb you have the altitude, numbring from 60/0 on the moveable piece toward the end. Ex. gr. At 7 deg. North declination, and 48 deg. Azimuth from South, the altitude is 35 deg. but at 7 deg. declination South, and 50 deg. Azimuth the altitude is onely 18 deg. 30 min.

PROBL. 14.

The altitude, declination, and right aſcenſion of any Star with the right aſcenſion of the Sun given, to find the hour of the night.

Take the Stars altitude from the Scale of altitudes; and laying the thread to his declination in the limb, find his hour from the laſt Meridian he was upon; as you did for the Sun by Probl. 5. If the Star be paſt the South, this is an afternoon hour; if not come to the South, a morning hour ; which keep.

Then ſetting one point of your Compaſſes in the Suns right aſcenſion (numbred upon the line twice 12 or 24 next the outward ledge on the fixed piece) extend the other point to the right aſcenſion of the Star numbred upon the ſame line, obſerving which way you turned the point of your Compaſſes, *viz.* toward the head or end. With this diſtance ſet one point of your Compaſſes in the Stars hour before found counted on the ſame line, and turning the other point the ſame way, as you did for the right aſcenſions, it gives the true hour of the night. Ex. gr. The 22 of *March* I find the altitude of the Lions heart 45 deg. his declination 13 d. 40 min. then by Probl. 5. I find his hour from the laſt Meridian 10 houres, 5 min. The right aſcenſion of the Sun is 46 m. of time, or 11 d. 30 m. of the Circle, the right aſcenſion of the Lions heart, is 9 hour 51 m. *ferè*, of time, or 147 deg. 43 m. of a circle ; then by a line of twice 12, you may find the true hour of the night, 7 hour 13 min.

PROBL. 15.

The right aſcenſion and declination of any Star, with the right aſcenſion of the Sun and time of night given, to find the altitude of that Star with his Azimuth from South, and by conſequent to find the Star, although before you knew it not.

This is no more than unravelling the laſt Problem. 1 Therefore upon the line of twice 12 or 24, ſet one point of your Compaſſes in the right aſcenſion of the Star, extending the other to the right aſcenſion of the Sun upon the ſame line, that diſtance laid the ſame way upon the ſame line, from the hour of the night, gives the Stars hour from the laſt Meridian he was upon. This found by Probl. 5. find his altitude as you did for the Sun. Laſtly, having now his declination and altitude by Probl. 8. or 10. according to his declination, you will ſoon get his Azimuth from South. This needs not an example.

By help of this Problem the Inſtrument might be ſo contrived, as to be one of the beſt Tutors for knowing of the Stars.

Probl. 16.

The altitude and Azimuth of any Star given to find his declination.

Lay the thread to the altitude counted on the limb from 60|0 on the moveable piece toward the end, ſetting one point of your Compaſſes in the Azimuth, take the neareſt diſtance to the thread; this applyed to the Scale of altitudes gives the declination. If the Azimuth given be on that ſide the thread toward the end, the declination is South; when on that ſide toward the head, its North.

Probl. 17.

The altitude and declination of any Star, with the right aſcenſion of the Sun, and hour of night given to find the Stars right aſcenſion.

By Probl. 5. or 14. find the Stars hour from the Meridian. Then on the line twice 12, or 24, ſet one point of your Compaſſes in the Stars hour (thus found) and extend the other to the hour of the night. Upon the ſame line with this diſtance ſet one point of your Compaſſes in the right aſcenſion of the

Sun, and turning the other point the ſame way, as you did for the hour, it gives the Stars right aſcenſion.

PROBL. 18.

The Meridian altitude given to find the time of Sun riſe and Sun ſet.

Take the Meridian altitude from your particular Scale, and ſetting one point of your Compaſſes upon the point 12 on the line of hours (that is the pin at the end) lay the thread to the other point, and on the line of hours the thread gives the time required.

PROBL. 19.

To find any latitude your particular Scale is made for.

Take the diſtance from 90, on the line of Azimuth unto the pin at the end of that line, or the point 12: this applyed to the particular Scale, gives the complement of that latitude the Inſtrument was made for.

PROBL. 20.

To find the angles of the ſubſtile, ſtile, inclination of Meridians, and ſix and twelve, for exact declining plains, in that latitude your Scale of altitudes is made for.

Sect. 1.

To find the diſtance of the ſubſtile from 12, *or the plains perpendicular.*

Lay the thread to the complement of declination counted on the line of Azimuths, and on the limb it gives the ſubſtile counting from 60/0 on the moveable piece.

Sect. 2.

To find the angle of the Stile's height.

On the line of Azimuths take the diſtance from the Plains declination to 90. This applyed to the Scale of altitudes gives the angle of the ſtile.

Sect. 3.

The angle of the Subſtile given to find the inclination of Meridians.

Take the angle of the ſubſtile from the Scale of altitudes, and applying it from 90 on the Azimuth line toward the end; the figures ſhew the complement of inclination of Meridians.

Sect. 4.

To find the angle betwixt 6 and 12.

Take the declination from the Scale of altitudes, and ſetting one point of your Compaſſes in 90 on the line of Azimuths, lay the thread to the other point and on the limb it gives the complement of the angle ſought, numbring from 60/0 on the moveable piece toward the end.

This laſt rule is not exact, nor is it here worth the labour to rectifie it by another line added; ſith you have an exact proportion for the Problem in the Treatiſe of Dialling *Chap.* 2. Sect. 5. Paragr. 4.

Chap. III.

Some uſes of the Line of natural ſigns on the Quadrantal ſide of the fixed piece.

Probl. I.

How to adde one ſign to another on the Line of Natural Sines.

TO adde one ſine to another, is to augment the line of one ſine by the line of the other ſine to be added to it. Ex. gr. To adde the ſine 15 to the ſine 20, I take the diſtance from the beginning of the line of ſines unto 15, and ſetting one point of the Compaſſes in 20, upon the ſame line, turn the other toward 90, which I finde touch in 37. So that in this caſe (for we regard not the Arithmetical, but proportional aggregate) 15 added to 20, upon the line of natural ſines, is the ſine 37 upon that line, and from the beginning of the line to 37 is the diſtance I am to take for the ſumme of 20 and 15 ſines.

PROBL. 2.

How to ſubſtract one ſine from another upon the line of natural ſines.

The ſubſtracting of one ſine from another, is no more than taking the diſtance from the leſſer to the greater on the line of ſines, and that diſtance applyed to the line from the beginning, gives the reſidue or remainer. Ex. gr. To ſubſtract 20 from 37 I take the diſtance from 20 to 37 that applyed to the line from the beginning gives 15 for the ſine remaining.

PROBL. 3.

To work proportions in ſines alone.

Here are four Caſes that include all proportions in ſines alone.

CASE I.

When the firſt term is Radius, or the Sine 90.

Lay the thread to the ſecond term counted on the degrees upon the moveable piece from the head toward the end, then numbring the third on the line of ſines, take the neareſt diſtance from thence to the thread, and that applyed to the Scale from the beginning gives the fourth term. Ex. gr.

As the Radius 90 is to the ſine 20, ſo is the ſine 30 to the ſine 10.

CASE 2.

When the Radius is the third term.

Take the ſine of the ſecond term in your Compaſſes, and enter it in the firſt term upon the line of ſines, and laying the thread to the neareſt diſtance, on the limb the thread gives the fourth term. Ex. gr.

As the ſine 30 is to the ſine 20, ſo is the Radius to the ſine 43. 30. min.

CASE 3.

When the Radius is the ſecond term.

Provided the third term be not greater than the firſt, tranſpoſe the terms. The method of tranſpoſition in this caſe is, as the firſt term is to the third, ſo is the ſecond to the fourth, and then the work will be the ſame as in the ſecond caſe. Ex. gr.

As the ſine 30 is to the radius or ſine 90, ſo is the ſine 20 to what ſine; which tranſpoſed is

As the ſine 30 is to the ſine 20, ſo is the radius to a fourth ſine, which will be found 43,30 min. as before.

CASE 4.

When the Radius is none of the three terms given.

In this caſe when both the middle terms are leſs than the firſt, enter the ſine of the ſecond term in the firſt, and laying the thread to the neareſt diſtance, take the neareſt extent from the third to the thread: this diſtance applyed to the ſcale from the beginning gives the fourth. Ex. gr.

As the ſine 20 to the ſine 10, ſo is the ſine 30 to the ſine 15.

When only the ſecond term is greater than the firſt, tranſpoſe the terms and work as before.

But when both the middle tearms be greater than the firſt, this proportion will not be performed by this line without a paralel entrance or double radius; which inconveniency ſhall be remedied in its proper place, when we ſhew how to work proportions by the lines of natural ſines on the proportional or ſector ſide.

Theſe four caſes comprizing the method of working all proportions by natural ſines alone, I ſhall propoſe ſome examples for the exerciſe of young practitioners, and therewith conclude this Chapter.

PROBL. 4.

To finde the Suns amplitude in any Latitude.

As the coſine of the Latitude is to the ſine of the Suns declination, ſo is the radius to the ſine of amplitude.

PROBL. 5.

To finde the hour in any Latitude in Northern Declination.

Proport. 1. As the radius to the ſine of the Suns declination, ſo is the ſine of the latitude to the ſine of the Suns altitude at ſix. By Probl. 2. ſubſtract this altitude at ſix from the preſent altitude, and take the difference. Then

Proport. 2. As the coſine of the latitude is to that difference, ſo is the radius to a fourth ſine. Again

Proport. 3. As the coſine of the declination to that fourth ſine, ſo is the radius to the ſine of the hour from ſix.

PROBL. 6.

To finde the hour in any Latitude when the Sun is in the Equinoctial.

As the coſine of the latitude is to the ſine of altitude, ſo is the radius to the ſine of the hour from ſix.

PROBL. 7.

To finde the hour in any latitude in Southern Declination.

Proport. 1. As the radius to the ſine of the Suns declination, ſo is the ſine of the latitude to the ſine of the Suns depreſſion at ſix; adde the ſine of depreſſion to the preſent altitude by Probl. 1. Then

Proport. 2. As the coſine of the latitude is to that ſumme, ſo is the radius to a fourth ſine. Again,

Proport. 3. As the coſine of declination is to the fourth ſine, ſo is the radius to the ſine of the hour from ſix.

PROBL.

PROBL. 8.

To finde the Suns Azimuth in any latitude in Northern Declination.

Proport. 1. As the ſine of the latitude to the ſine of declination, ſo is the radius to the ſine of altitude at Eaſt, or Weſt. By Probl.2. ſubſtract this from the preſent altitude, then,

Proport. 2. As the coſine of the latitude is to that reſidue, ſo is the radius to a fourth ſine. Again,

Proport. 3. As the coſine of the altitude is to that fourth ſine, ſo is the radius to the ſine of the Azimuth from Eaſt or Weſt.

PROBL. 9.

To finde the Azimuth for any latitude when the Sun is in the Equator.

Proport. 1. As the coſine of the latitude to the ſine of altitude, ſo is the ſine of the latitude to a fourth ſine.

Proport. 2. As the coſine of altitude to that fourth ſine, ſo is the radius to the ſine of the Azimuth from Eaſt, or Weſt.

PROBL. 10.

To finde the Azimuth for any latitude in Southern Declination.

Proport. 1. As the coſine of the latitude to the ſine of altitude, ſo is the ſine of the latitude to a fourth. Having by Probl. 4. found the Suns amplitude, adde it to this fourth ſine by Probl. 1. and ſay

As the coſine of the altitude is to the ſum, ſo is the radius to the ſine of the Azimuth from Eaſt or Weſt. The terms mentioned in the 5th. 7th. 8th. 10th. Problems are appropriated unto us that live on the North ſide the Equator. In caſe they be applyed to ſuch latitudes as lie on the South ſide the Equator. Then what is now called Northern declination, name Southern, and what is here ſtyled Southern declination, term Northern, and all the proportion with the operation is the ſame.

These proportions to finde the hour and Azimuth, may be all readily wrought by the lines of artificial ſines, only the addition and ſubſtraction muſt alwayes be wrought upon the line of natural ſines.

Chap. IV.

Some uſes of the Lines on the proportional ſide of the Inſtrument, viz. *the Lines of natural Sines, Tangents, and Secants.*

Probl. I.

To lay down any Sine, Tangent, or Secant to a Radius given. See Fig. 1.

IF you be to lay down a Sine, enter the Radius given in 90, and 90 upon the lines of Sines, keeping the Sector at that gage, ſet one point of your Compaſſes in the Sine required upon one line, and extend the other point to the ſame Sine upon the other Line: This diſtance is the length of the Sine required to the given Radius. Ex. gr. Suppoſe A. B. the Radius given, and I require the Sine 40. proportional to that Radius. Enter A. B. in 90, and 90 keeping the Sector at that gage, I take the diſtance 'twixt 40 on one ſide, to 40 on the other, that is, C. D. the Sine required.

The work is the ſame, to lay down a Tangent to any Radius given, provided you enter the given Radius in 45, and 45, on the line of Tangents. Only obſerve if the Tangent required be leſs than 45. you muſt enter the Radius in 45. and 45 next the end of the Rule. But when the Tangent required exceeds 45. enter the Radius given in 45, and 45 'twixt the center and end, and keeping the Sector at that Gage, take out the Tangent required. This is ſo plain, there needs no example.

To lay down a Secant to any Radius given, is no more than to enter the Radius in the two pins at the beginning of the line of Secants, and keeping the Sector at that Gage, take the diſtance from the number of the Secant required on one ſide, to the ſame number on the other ſide, and that is the Secant ſought at the Radius given.

The uſe of this Problem will be ſufficiently ſeen in delineating Dyals, and projecting the Sphere.

PROBL. 2.

To lay down any Angle required by the Lines of Sines, Tangents, and Secants. See Fig. 2.

There are two wayes of protracting an Angle by the Line of Sines, Firſt if you uſe the Sines in manner of Chords. Then having drawn the line A B at any diſtance of your Compaſs, ſet one point in B, and draw a mark to interſect the Line B A, as E F. Enter this diſtance B F in 30, and 30 upon the Lines of Sines, and keeping the Sector at that Gage, take out the Sine of half the Angle required, and ſetting one point where F interſects B A, turn the other toward E, and make the mark E, with a ruler draw B E and the Angle E B F is the Angle required, which here is 40. d.

A ſecond method by the lines of Sines is thus, Enter B A Radius in the Lines of Sines, and keeping the Sector at that Gage, take out the Sine of your Angle required with that diſtance, ſetting one point of your Compaſſes in A, ſweep the ark D, a line drawn from B by the connexity of the Ark D, makes the Angle A B C 40 d. as before.

To protract an Angle by the Lines of Tangents is eaſily done, draw B A the Radius upon A, erect a perpendicular, A C, enter B A in 45, and 45 on the Lines of Tangents, and taking out the Tangent required (as here 40) ſet it from A to C. Laſtly, draw B C, and the Angle C B A is 40 d. as before.

In caſe you would protract an Angle by the Lines of Secants. Draw B A, and upon A erect the perpendicular A C, enter A B in the beginning of the Lines of Secants, and take out the Secant of the Angle, with that diſtance, ſetting one point of your Compaſſes in B, with the other croſs the perpendicular A C, as in C. This done, lay a Ruler to B, and the point of interſection, and draw the Line B C. So have you again the Angle C B A. 40. d. by another projection. Theſe varieties are here inſerted only to ſatisfie a friend, and recreate the young practitioner in trying the truth of his projection.

PROBL. 3.

To work proportions in Sines alone, by the Lines of natural Sines on the proportional ſide of the Inſtrument.

The general rule is this. Account the firſt term upon the Lines of Sines from the Center, and enter the ſecond term in the firſt ſo accounted, keeping the Sector at that Gage, account the third term on both lines from the Center, and taking the diſtance from the third term on one line to the third term on the other line, meaſure it upon the line of Sines from the beginning, and you have the fourth term. Ex. gr.

As the Radius is to the Sine 30, ſo is the Sine 40 to the Sine 18. 45.

There is but one exception in this Rule, and that is when the ſecond term is greater than the firſt; yet the third leſſer than the firſt, and in this caſe tranſpoſe the terms, by Chap 3. Probl. 3. Caſe 3.

But when the ſecond term is not twice the length of the firſt, it may be wrought by the general Rule without any tranſpoſition of terms. Ex. gr.

As the Sine 30 is to the Sine 50, ſo is the Sine 20 to the Sine 31. 30. min.

And by conſequent, when the third term is greater than the firſt, provided it be not upon the line, double the length thereof, it may be wrought by tranſpoſing the terms, although the ſecond was twice the length of the firſt. Ex. gr.

As the Sine 20 is to the Sine 60, ſo is the Sine 42, to what Sine? which tranſpoſed is,

As the Sine 20 is to the Sine 42, ſo is the Sine 60 to the Sine 35. 30.

This caſe will remove the inconveniency mentioned, Chap. 3. Probl. 3. Caſe 4. of a double Radius. I intended here to have adjoyned the method of working proportions by natural Tangents alone, and by natural Sines, and Tangents, conjunctly: But conſidering the multiplicity of proportions when the Tangents exceed 45. I ſuppoſe it too troubleſome for beginners, and a needleſs variety for thoſe that are already Mathematicians. Sith, both may be eaſed by the artificial Sines and Tangents on the outward ledge, where I intend to treat of thoſe Caſes at large, and ſhall in this place only annex ſome proportions in Sines alone, for the exerciſe of young beginners.

PROBL. 4.

By the Lines of Natural Sines to lay down any Tangent, or Secant required to a Radius given.

In ſome Caſes, eſpecially for Dyalling, your Inſtrument may be defective of a Tangent, or Secant for your purpoſe, Ex. gr. when the Tangent exceeds 76, or the Secant is more than 60. In theſe extremities uſe the following Remedies. Firſt, for a Tangent.

As the coſine of the Ark is to the Radius given, ſo is the ſine of the Ark to the length of the Tangent required. Secondly, for a Secant.

As the coſine of the Ark is to the Radius given, ſo is the Sine 90 to the length of the Secant required.

PROBL. 5.

The diſtance from the next Equinoctial Point given to finde the Suns declination.

As the Radius to the ſine of the Suns greateſt declination, ſo is the ſine of his diſtance from the next Equinoctial Point to the ſine of his preſent declination.

PROBL. 6.

The declination given to finde the Suns Equinoctial Diſtance.

As the ſine of the greateſt declination is to the ſine of the preſent declination, ſo is the Radius to the ſine of his Equinoctial Diſtance.

PROBL. 7.

The Altitude, Declination, and Diſtance of the Sun from the Meridian given to finde his Azimuth.

As the coſine of the altitude, to the coſine of the hour from the Meridian, ſo is the coſine of declination to the ſine of the Azimuth.

Chap. V.

Some uſes of the Lines of Lines, on the proportional ſide of the Inſtrument.

Probl. I.

To divide a Line given into any Number of equal parts. See Fig. 3.

SUppoſe A B a Line given to be divided into nine equal parts. Enter A B in 9, and 9 on the lines of lines, keeping the Sector at that gage, take the diſtance from 8, on one ſide, to 8 on the other, and apply it from A upon the line A B, which reacheth to C; then is C B a ninth part of the line A B. By this means you may divide any line (that is not more than the Inſtrument in length) into as many parts as you pleaſe, *viz.* 10, 20, 30, 40, 50, 100, 500, *&c.* parts according to your reckoning the diviſions upon the lines, Ex. gr. The line is actually divided into 200 parts, *viz.* firſt into 10, marked with Figures, and each of thoſe into twenty parts more. Again, if the line repreſents a 1000, then every figured diviſion is 100, the ſecond or ſhorter diviſion is 10, and the third or ſhorteſt diviſion is 5. In caſe the whole line was 2000, then every figured diviſion is 200, every ſmaller or ſecond diviſion is 20, every third or ſmalleſt diviſion is 10, *&c.* Suppoſe I have any line given, which is the baſe of a Triangle, whoſe content is 2000 poles, and I demand ſo much of the Baſe as may anſwer 1750 poles. Enter the whole line in 10, and 10 at the end of the lines of lines, and keep the Sector at that gage. Now the whole line repreſenting 2000 poles, every figured diviſion is 200; therefore 1700 is eight and an half of the figured diviſions, and 50 is five of the ſmalleſt diviſions more (for in this caſe every ſmalleſt diviſion is 10, as was before expreſſed) wherefore ſetting one point of the Compaſſes in 15 of the ſmalleſt diviſions beyond 8 on the Rule. I extend the other point to the ſame diviſion upon the line on the other ſide, and that diſtance is 1750 poles in the baſe of the Triangle propoſed. How ready this is to ſet out a juſt quantity in any plat of ground, I ſhall ſhew in a Scheam, Chap. 12.

PROBL. 2.

To work proportions in Lines, or Numbers, or the Rule of three direct by the Lines of Lines.

Enter the ſecond term in the firſt, and keeping the Sector at that gage, take the diſtance 'twixt the third on one line, to the third on the other line, that diſtance is the fourth in lines, or meaſured upon the line from the centre, gives the fourth in numbers, Ex. gr.

As 7 is to 3, ſo is 21 to 9.

PROBL. 3.

To work the Rule of Three inverſe, or the back Rule of Three by the Lines of Lines.

In theſe proportions there are alwayes three terms given to finde a fourth, and of the three given terms two are of one denomination (which for diſtinction ſake I call the double denomination;) and the third term is of a different denomination from thoſe two, which I therefore call the ſingle denomination, of which the fourth term ſought muſt alſo be. Now to bring theſe into a direct proportion, the rule is this. When the fourth term ſought is to be greater than the ſingle denomination (which you may know by ſight of the terms given) ſay,

As the leſſer double denomination is to the greater double denomination, ſo is the ſingle denomination to the fourth term ſought. The work is by Probl. 2.

If 60 men do a work in 5 dayes, how long will 30 men be about it?

As 30 is to 60, ſo is 5 to 10. The number of dayes for 30 men in the work.

Again, when the fourth term is to be leſs than the ſingle denomination, ſay,

As the greater double denomination is to the leſſer double denomination, ſo is the ſingle denomination to the fourth term ſought.

If 30 men do a work in 5 dayes, how long ſhall 60 be doing of it?

As 60 is to 30, ſo is 5 to $2\frac{1}{2}$. The time for 60 men in the work.

PROBL. 4.

The length of any perpendicular, with the length of the ſhadow thereof given, to finde the Suns Altitude.

At the length of the ſhadow upon the lines of lines, is to the Tangent 45, ſo is the length of the perpendicular numbred upon the lines of lines, to the tangent of the Suns altitude.

PROBL. 5.

To finde the Altitude of any Tree, Steeple, &c. at one ſtation.

At any diſtance from the object (provided the ground be level) with your Inſtrument, look to the top of the object along the outward ledge of the fixed piece, and take the angle of its altitude. This done, meaſure by feet or yards, the diſtance from your ſtanding to the bottom of the object. Then ſay,

As the coſine of the altitude is to the meaſured diſtance numbred upon the lines of lines, ſo is the ſine of the altitude to a fourth number of feet or yards (according to the meaſure you meeted the diſtance) to this fourth, adde the height of your eye from the ground, and that ſum gives the number of feet or yards in the altitude.

CHAP. VI.

How to work proportions in Numbers, Sines, or Tangents, by the Artificial Lines thereof on the outward ledge.

THe general rule for all of theſe, is to extend the Compaſſes from the firſt term to the ſecond (and obſerving whether that extent was upward or downward) with the ſame diſtance, ſet one point in the third term, and turning the other point the ſame way, as at firſt, it gives the fourth. But in Tangents when any of the terms exceeds 45, there may be excurſions, which in their due place I ſhall remove.

PROBL. I.

Numeration by the Line of Numbers.

The whole line is actually divided into

to 100 proportional parts, and accordingly diſtinguiſhed by figures, 1, 2, 3, 4, 5, 6, 7, 8, 9, and then, 10, 20, 30, 40, 50, 60, 70, 80, 90, 100. So that for any number under 100, the Figures readily direct you, Ex. gr. To finde 79 on the line of numbers, count 9 of the ſmall diviſions beyond 70, and there is the point for that number. Now as the whole line is actually divided into 100 parts, ſo is every one of thoſe parts ſubdivided (ſo far as conveniency will permit) actually into ten parts more, by which means you have the whole line actually divided into 1000 parts. For reckoning the Figures impreſſed, 1, 2, 3, 4, 5, 6, 7, 8, 9, to be 10, 20, 30, 40, 50, 60, 70, 80, 90, and the other figures which are ſtamped 10, 20, 30, 40, 50, 60, 70, 80, 90, 100, to be 100, 200, 300, 400, 500, 600, 700, 800, 900, 1000. You may enter any number under 1000 upon the line, according to the former directions. And any numbers whoſe product ſurmount not 1000, may be wrought upon this line; but where the product exceeds 1000, this line will do nothing accurately: Whereſore I ſhall willingly omit many Problems mentioned by ſome Writers to be wrought by this line, as ſquaring, and cubing of numbers, &c. Sith they have only nicety, and nothing of exactneſs in them.

PROBL. 2.

To multiply two numbers, given by the Line of Numbers. The proportion is this.

As 1 on the line is to the multiplicator, ſo is the multiplicand to the product. Ex. gr.

As 1 is to 4, ſo is 7, to what?

Extend the Compaſſes from the firſt term, *viz.* 1 unto the ſecond term, *viz.* 4. with that diſtance, ſetting one point in 7 the third term, turn the other point of the Compaſſes toward the ſame end of the rule, as at firſt, and you have the fourth, *viz.* 28. There is only one difficulty remaining in this Problem, and that is to determin the number of places, or figures in the product, which may be reſolved by this general rule. The product alwayes contains as many figures as are in the multiplicand, and multiplicator both, unleſs the two firſt figures of the product be greater than the two firſt figures in the multiplicator, and then the product muſt have one figure leſs than are in the multiplicator, and multiplicand both. Ex. gr. 47 multiplied by 25, is 2175, conſiſting of four figures; but 16 multiplied by 16, is 240, conſiſting of no more than three places, for the reaſon before mentioned.

tioned. I here (for diſtinction ſake) call the multiplicator the leſſer of the two nnmbers, although it may be either of them at pleaſure.

PROBL. 3.

To work Diviſion by the Line of Numbers.

As the diviſor is to 1, ſo is the dividend to the quotient.

Suppoſe 800 to be divided by 20, the quotient is 40. For,

As 20 is to 1, ſo is 800 to 40.

To know how many figures you ſhall have in the quotient, take this rule. Note the difference of the numbers of places or figures in the dividend and diviſor. Then in caſe the quantity of the two firſt figures to the left hand in your diviſor be leſs than the quantity of the two firſt figures to the left hand in your dividend, the quotient ſhall have one figure more than the number of difference: But where the quantity of the two firſt figures of the diviſor is greater than the quantity of the two firſt figures of the dividend, the quotient will have only that number of figures noted by the difference. Ex. gr. 245 divided by 15. will have two figures in the quotient;

but 16 divided b ave only one figure in the quotient.

Probl. 4.

To finde a mean proportional 'twixt two Numbers given by the line of Numbers.

Divide the ſpace betwixt them upon the line of numbers into two equal parts, and the middle point is the mean proportional. Ex. gr. betwixt 4 and 16, the mean proportional is 8. If you were to finde two mean proportionals, divide the ſpace 'twixt the given numbers into three parts. If four mean proportionals divide it into five parts, and the ſeveral points 'twixt the two given numbers, will ſhow the reſpective mean proportionals.

Probl 5.

To work proportions in Sines alone, by the Artificial Line of Sines.

Extend the Compaſſes from the firſt term to the ſecond, with that diſtance ſet one point in the third term, and the other point gives the fourth. Only obſerve that if the ſecond term be leſs than the firſt, the fourth muſt be leſs tl [illegible], or if the ſecond term exceed t[illegible] fourth will be greater than the third. This may direct you in all proportions of ſines and tangents ſingly or conjunctly, to which end of the rule to turn the point of your Compaſſes, for finding the fourth term. Ex. gr.

As the ſine 60 is to the ſine 40, ſo is the ſine 20 to the ſine 14. 40. Again,

As the ſine 10 is to the ſine 20, ſo is the ſine 30 to the ſine 80.

PROBL. 6.

To work proportions in Tangents alone by the Artificial Line of Tangents.

For this purpoſe the artificial line of tangents muſt be imagined twice the length of the rules, and therefore for the greater conveniency, it is doubly numbred, *viz.* Firſt from 1 to 45, which is the radius, or equal to the ſine 90: In which account every diviſion hath (as to its length on the rule) a proportional decreaſe. Secondly, its numbred back again from 45 to 89, in which account every diviſion hath (as to its length on the line) a proportional encreaſe. So that the tangent 60 you muſt imagine the whole length of the Rule; and ſo much more as the diſtance from 45 unto 30 or 60 is. This well obſerved, all proportions in tangents are wrought after the ſame manner of extending the Compaſſes from the firſt term to the ſecond, and that diſtance ſet in the third, gives the fourth, as was for ſines and numbers. But for the remedying of excurſions, ſith the line is no more than half the length, we muſt imagine it. I ſhall lay down theſe Caſes.

CASE 1.

When the fourth term is a tangent exceeding 45, or the Radius. Ex. gr.

As the tangent 10 is to the tangent 30, ſo is the tangent 20, to what?

Extending the Compaſſes from 10 on the line of tangents to 30, with that diſtance I ſet one point in 20, and finde the other point reach beyond 45, which tells me the fourth term exceeds 45, or the radius; wherefore with the former extent, I ſet one point in 45; and turning the other toward the beginning of the line, I mark where it toucheth, and from thence taking the diſtance to the third term, I have the exceſs of the fourth term above 45 in my compaſs: wherefore with this laſt diſtance ſetting one point in 45, I turn the other upon the line, and it reacheth to 50, the tangent ſought.

CASE 2.

When the firſt term is a tangent exceeding 45, *or the Radius. Ex. gr.*

As the tangent 50 is to the tangent 20, ſo is the tangent 30. to what?

Becauſe the ſecond term is leſs than the firſt, I know the fourth muſt be leſs than the third. All the difficulty is to get the true extent from the tangent 50 to 20. To do this, take the diſtance from 45 to 50, and ſetting one point in 20, the ſecond term, turn the other toward the beginning of the line, marking where it toucheth, extend the Compaſſes from the point where it toucheth to 45, and you will have the ſame diſtance in your Compaſſes as from 50 to 20, if the line had been continued at length unto 89 tangents, with this diſtance, ſet one point in 30 the third term, and turn the other toward the beginning (becauſe you know the fourth muſt be leſs) and it gives 10 the tangent ſought.

CASE. 3.

When the third term is a Tangent exceeding 45, or the Radius.

As the tangent 40 is to the tangent 12, 40 min. ſo is the tangent 65, to what?

Extend the Compaſſes from 40 to 12 d. 40 min. with this diſtance, ſetting one point in 65. turn the other toward 45, and you will finde it reach beyond it, which aſſures you the fourth term will be leſs than 45. Therefore lay the extent from 45 toward the beginning, and mark where it toucheth, take the diſtance from that point to 65, and laying that diſtance from 45 toward the beginning it gives 30, the tangent ſought.

Theſe Caſes are ſufficient to remove all difficulties. For when the ſecond term exceeds the Radius, you may tranſpoſe them, ſaying, as the firſt term is to the third ; ſo is the ſecond to the fourth, and then its wrought by the third Caſe.

I ſuppoſe it needleſs to adde any thing about working proportions by ſines and tangents conjunctly, ſith, enough hath been already ſaid of both of them apart, in theſe two laſt Problems ; and the work is the ſame when they are intermixed. Only ſome proportions I ſhall adjoyn, and leave to the practice of the young beginner, with the directions in the former Caſes.

PROBL. 7.

To finde the Suns aſcenſional difference in any Latitude.

As the co-tangent of the latitude is to the tangent of the Suns declination, ſo is the radius to the ſine of the aſcenſional difference.

PROBL. 8.

To finde at what hour the Sun will be Eaſt, or Weſt in any Latitude.

As the tangent of the latitude is to the tangent of the Suns declination, ſo is the radius to the coſine of the hour from noon.

PROBL. 9.

The Latitude, Declination of the Sun, and his Azimuth from South, given to finde the Suns Altitude at that Azimuth.

As the radius to the coſine of the Azimuth from South, ſo is the co-tangent of the latitude, to the tangent of the Suns altitude in the equator at the Azimuth given. Again,

As the ſine of the latitude is to the ſine of the Suns declination, ſo is the coſine of the Suns altitude in the equator (at the ſame Azimuth from Eaſt or Weſt) to a fourth ark.

When the Azimuth is under 90, and the latitude and declination is under the ſame pole, adde this fourth ark to the altitude in the equator. In Azimuths exceeding 90, when the latitude and declination is under the ſame pole, take the equator altitude out of the fourth ark. Laſtly, when the latitude and declination reſpect different poles, take the fourth ark out of the equator altitude, and you have the altitude ſought.

PROBL. 10.

The Azimuth, Altitude, and Declination of the Sun, given to finde the hour.

As the coſine of declination is to the ſine of the Suns Azimuth, ſo is the coſine of the altitude to the ſine of the hour from the Meridian.

Proportions may be varied eight ſeveral wayes in this manner following.

1. As the firſt term is to the ſecond, ſo is the third to the fourth.

2. As the ſecond term is to the firſt, ſo is the fourth to the third.

3. As the third term is to the firſt, ſo is the fourth to the ſecond.

4. As the fourth term is to the ſecond, ſo is the third to the firſt.

5. As the ſecond term is to the fourth, ſo is the firſt to the third.

6. As the firſt term is to the third, ſo is the ſecond to the fourth.

7. As the third term is to the fourth, ſo is the firſt to the ſecond.

8. As the fourth term is to the third, ſo is the ſecond to the firſt.

By theſe any one may vary the former proportions, and make the Problems three times the number here inſerted. Ex. gr. To finde the aſcenſional difference in Problem 10, of this Chapter, which runs thus.

As the co-tangent of the latitude is to the tangent of the Suns declination, ſo is the radius to the ſine of aſcenſional difference. Then by the third variety you may make another Problem, viz.

As the radius is to the co-tangent of the latitude, ſo is the ſine of the Suns aſcenſional difference to the ſine of his declination. Again, by the fourth variety you may make a third Problem, thus, As the ſine of the Suns aſcenſional difference is to the tangent of the Suns declination, ſo is the radius to the co-tangent of the latitude.

By this Artifice many have ſtuffed their Books with bundles of Problems.

CHAP. VII.

Some uſes of the Lines of Circumference, Diameter, Square Equal, and Square Inſcribed.

ALL theſe are lines of equal parts, bearing ſuch proportion to each other, as the things ſignified by their names. Their uſe is this; Any one of them given in inches or feet, *&c.* to finde how much any of the other three are in the ſame meaſure. Suppoſe I have the circumference of a Circle, Tree, or Cylinder given in inches, I take the ſame number of parts (as the Circle is inches) from the line of circumference, and applying that diſtance to the reſpective lines, I have immediately the ſquare equal, ſquare inſcribed, and diameter, in inches, and the like, if any of thoſe were given to finde the circumference. This needs no example.

The conveniency of this line any one may experiment in ſtanding timber; for taking the girth, or circumference with a line, finde the diameter; from that diameter abate twice the thickneſs of the bark, and you have the true diameter, when its barked, and by Chap. 9. Probl. 5. you will gueſs very near at the quantity of timber in any ſtanding Tree.

CHAP. VIII.

To meaſure any kinde of Superficies, as Board, Glaſs, Pavement, Walnſcot, Hangings, Walling, Slating, or Tyling, by the line of Numbers on the outward ledge.

THe way of accounting any number upon, or working proportions by, the line of numbers, is ſufficiently ſhewn already, Chap. 6. which I ſhall not here repeat, only propoſe the proportions for theſe Problems, and refer you to thoſe directions.

PROBL. I.

The breadth of a Board given in inches, to finde how many inches in length make a foot at that breadth, ſay,

As the breadth in inches is to 12, ſo

is 12 to the length in inches for a foot at that breadth. Ex. gr. At 8 inches breadth you muſt have 18 inches in length for a foot.

PROBL. 2.

The breadth and length of a Board given to finde the content.

As 12 is to the length in feet and inches, ſo is the breadth in inches to the content in feet. Ex. gr. at 15 inches breadth, and 20 foot length, you have 25 foot of Board.

PROBL. 3.

A ſpeedy way to meaſure any quantity of Board.

The two former Problems are ſufficient to meaſure ſmall parcels of Board, When you have occaſion to meaſure greater quantities, as 100 foot, or more, lay all the boards of one length together, and when the length of the boards exceeds 12 foot, uſe this proportion.

As the length in feet and inches is to 12, ſo is 100 to the breadth in inches for an 100 foot. Ex. gr. At 30 foot in length 40 inches in breadth, make an 100 foot of board (reckoning five ſcore to the hundred.) This found with a rule or line, meaſure 40 inches at both ends in breadth, and you have 100 foot. When one end is broader than another, you may take the breadth of the over-plus of 100 foot at both ends, and taking half that ſum for the true breadth of the over-plus by Probl. 2. finde the content thereof.

When your boards are under 12 foot in length, ſay,

As the length in feet and inches is to 12, ſo is 50 unto the breadth in inches for 50 foot of board, and then you need only double that breadth to meaſure 100 foot as before. In like manner you may meaſure two, three, four, five, a hundred, *&c.* foot of board ſpeedily, as your occaſion requires.

PROBL. 4.

To meaſure Wainſcot, Hangings, Plaiſter, &c.

Theſe are uſually computed by the yard, and then the proportion is.

As nine to the length in feet, and inches, ſo is the breadth or depth in feet and inches to the content in yards. Ex. gr. At 18 foot in length, and two foot in breadth, you have four yards.

PROBL. 5.

To meaſure Maſons, or Slaters Work, as Walling, Tyling, &c.

The common account of theſe is by the rood, which is eighteen foot ſquare, that is 324 ſquare foot in one rood, and then the proportion is.

As 324 to the length in feet, ſo is the breadth in feet to the content in roods.

Ex. gr. At 30 foot in length, and 15 foot in breadth, you have 1 rood $\frac{3}{10}$ and better, or one rood $\frac{126}{324}$ parts of a rood.

CHAP. IX.

The menſuration of Solids, as Timber, Stone, &c. *by the lines on the proportional ſide of the looſe piece.*

THeſe two lines meeting upon one line in the midſt betwixt them (for diſtinction ſake) I call one the right, the other the left line, which are known by the hand they ſtand toward when you hold up the piece in the right way to read the Figures.

The right line hath two figured partitions. The firſt partition is from 3, at the beginning to the letters Sq. every figured diviſion repreſenting an inch, and each ſubdiviſion quarters of an inch.

The next partition is from the letters Sq. unto 12, at the end, every figured diviſion ſignifying a foot, and each ſub-diviſion the inches in a foot. The letters T R, and T D, are for the circumference and Diameter in the next meaſuring of Cylinders. The letters R and D. for the meaſuring of Timber, according to the vulgar allowance, when the fourth part of the girt is taken, *&c.* The letters A and W are the gauger points for Ale and Wine Meaſures. Laſtly, the figures 12 'twixt D and T D. are for an uſe, expreſſed Probl. 2.

The left line alſo hath two figured partitions, proceeding firſt from 1 at the beginning to one foot, or 12 inches, each whereof is ſub-divided into quarters. From thence again to 100, each whereof to 10 foot, is ſub-divided into inches, *&c.* and every foot is figured. But from 10 foot to 100, only every tenth foot is figured; the ſub-diviſions repreſenting feet. The method of working proportions by theſe lines (only obſerving the ſides) is the ſame as by the line of Numbers, *viz.* extending from the firſt to the ſecond, *&c.*

PROBL. I.

To reduce Timber of unequal breadth and depth to a true Square.

As the breadth on the left is to the breadth on the right, ſo is the depth on the left, to the ſquare on the right line.

At 7 inches breadth, and 18 inches depth, you have 11 inches $\frac{1}{4}$ and better for the true ſquare.

PROBL. 2.

The ſquare of a Piece of Timber given in Inches, or Feet, and Inches to finde how much in length makes a Foot.

As the ſquare in feet and inches on the right is to one foot on the left, ſo is the point Sq. on the right to the number of feet and inches on the left for a foot ſquare of Timber.

At 18 inches ſquare, 5 inches $\frac{1}{4}$ and almoſt half a quarter in length makes a foot. When your Timber (if it be proper to call ſuch pieces by that name) is under 3 inches ſquare, account the figured diviſions on the right line from the letters Sq. to the end, for inches, and each ſub-diviſion twelve parts of an inch. So that every three of them makes a quarter of an inch. Then the proportion is, as the inches and quarters ſquare on the right is to 100 on the left, ſo is the point 12 'twixt D, and T D, on the right, to the number of feet in length on the left to make a foot of Timber.

As 2 inches ½ ſquare you muſt have 23 foot 6 inches, and ſomewhat better for the length of a foot of Timber.

PROBL. 3.

The ſquare and length of a piece of Timber given to finde the content.

As the point Sq. on the right is to the length in feet and inches on the left, ſo is the ſquare in feet and inches on the right to the content in feet on the left.

At 30 foot in length, and 15 inches ſquare, you have 46 foot ½ of Timber.

At 20 foot in length, and 11 inches ſquare you have 16 foot, and almoſt ¾ of Timber.

When you have a great piece of Timber exceeding 100 foot (which you may eaſily ſee by the excurſion upon the rule) then take the true ſquare, and half the length, finde the content thereof by the former proportion, and doubling that content, you have the whole content.

PROBL. 4.

The Circumference, or girth, of a round piece of Timber, being given, together with the length, to finde the content.

As the point R. on the right to the length in feet and inches on the left, ſo is the circumference in feet and inches on the right, to the content in feet on the left.

At 20 foot in length, and 7 foot in girth, you have 60 foot of Timber for the content. This is after the common allowance for the waſte in ſquaring; and although ſome are pleaſed to quarrel with the allowancer, as wronging the ſeller, and giving the quantity leſs, than in truth it is; yet I preſume when they buy it themſelves, they ſcarcely judge thoſe Chips worth the hewing, and have as low thoughts of the over-plus, as others have of that their admonition.

If it be a Cylinder that you would take exact content of, then ſay,

As the point T R, on the right, is to the length on the left, ſo is the girt on the right to the exact content on the left.

At 15 foot in length, and 7 foot in girt, you have 59 foot of ſolid meaſure.

The Diameter of any Cylinder given, you may by the ſame proportion finde the content, placing the point D, inſtead of R, in the proportion for the uſual allowance, and the point T D, for the exact compute.

PROBL. 5.

To meaſure tapered Timber.

Take the ſquare or girt at both ends, and note the ſum and difference of them. Then for round Timber, as the point R. on the right, is to the length on the left, ſo is half the ſum of the girt at both ends on the right, to a number of feet on the left. Keep this number, and ſay again,

As the point R. on the right, is to the third part of the former length on the left, ſo is half the difference of the girts on the right, to a number of feet on the left; which number added to the former, gives the true content. The ſame way you may uſe for ſquare Timber, only ſetting the feet and inches ſquare inſtead of the girt, and the point Sq. inſtead of the point R. At 30 foot in length, 7 foot at one end, and 5 at the other in girt, half the ſum of the girts is 6 foot, or 72 inches; the firſt number of feet found 67, half the difference of the girts is 1 foot, or 12 inches, the third part of the length 10 foot; then the ſecond number found will be 7 foot, one quarter and half a quarter. The ſum of both (or true content) 74 foot, one quarter, and half a quarter.

For ſtanding Timber, take the girt about a yard from the bottom, and at 5 foot from the bottom, by Chap. 7, ſet down theſe two diameters without the bark; and likewiſe the difference 'twixt them. Again, by Chap. 6. Probl. 4. finde the altitude of the tree, ſo far as it bears Timber (or as we commonly phraſe it, to the collar) this done, you may very near proportion the girt at the collar and content of the tree, before it falls.

In caſe any make choice of the hollow contrivance mentioned, Chap. 1. they need no compaſſes in the menſuration of any ſolid; provided the lines for ſolid meaſure, and gauging veſſels, be doubly impreſſed (only in a reverted order, one pair of lines proceeding from the head toward the end, and the other pair from the end toward the head) upon the ſliding cover, and its adjacent ledges.

This done, the method of performing any of the Problems mentioned in this Chapter, is eaſie. For whereas you are before directed to extend the Compaſſes from the firſt term to the ſecond; and with that diſtance ſetting one point in the third term, the other point gave the fourth, or term ſought. So here, obſerving the lines as before, ſlide the cover until the firſt term ſtand directly againſt the ſecond; then looking for the third on its proper line, it ſtands exactly againſt the fourth term, or term ſought on the other line.

Only note, that when the ſecond term is greater than the firſt, its performed by that pair of lines proceeding from the head toward the end: But when the firſt term is greater than the ſecond, it is reſolved by that pair of lines which is numbred from the end toward the head.

CHAP. X.

To Gauge Veſſels, either for Wine, or Ale, Meaſure.

PROBL. I.

The Diameter at Head, and Diameter at Boung, given in Inches, and tenth parts of an Inch, to finde the mean Diameter in like meaſure.

TAke the difference in inches, and tenth parts of an inch, between the two diameters. Then ſay by the line of numbers,

As 1 is to 7, ſo is the difference to a fourth number of inches, and tenth parts of an inch. This added to the Diameter at head, gives the mean Diameter. Ex. gr.

At 27 inches the boung, and 19 inches two tenths at the head, the difference is 7 inches, 8 tenths. The fourth number found by the proportion will be 5 inches, 4 tenths, and one half, which added to the diameter at the head, gives 24 inches, 6 tenths, and one half tenth of an inch for the mean diameter.

PROBL. 2.

The length of the Veſſel, and the mean Diameter given in Inches, and tenth parts of an Inch, to finde the content in Gallons, either in Wine or Ale meaſure.

Note firſt, that the point A, on the right is the gauge point for Ale meaſure, and the point W. on the right, is the gauge point for Wine meaſure. Then ſay,

As the gauge point on the right to the length in inches, and tenth parts of an inch on the left, ſo is the mean diameter in inches and tenth parts of an inch on the right, to the content in gallons on the left.

CHAP. XI.

Some uſes of the Lines on the inward ledge of the moveable piece.

THe Line of inches and foot meaſure do by inſpection (only) reduce either of the meaſures into the other. Ex. gr. Three on the line of inches ſtands directly againſt 25 cents of foot meaſure. Or 75 cents of foot meaſure is directly againſt 9 on the inches.

Another uſe of theſe lines (welcome perhaps to them that delight in inſtrumental computations) is to know the price of carriage for any quantity, *&c.* by inſpection.

For this purpoſe, the line of inches repreſents the price of a pound, every inch being a penny, and every quarter a farthing. The prickt line is the price of an hundred pound at five ſcore and twelve to the hundred, every diviſion ſignifying a ſhilling.

The line of foot meaſure is the price of an hundred pound at five ſcore to the hundred, every diviſion ſtanding for a ſhilling. Ex. gr.

At 3 pence the pound on the inches, is 28 ſhillings on the prickt line, and 25 ſhillings on the foot meaſure, for the price of an hundred, the like for the converſe. Otherwiſe, the price of a pound being given, the rate of an hundred is readily computed without the rule. For conſidering the number of farthings is in the price of a pound, twice that number of ſhillings, and once that number of pence, is the price of an hundred, reckoning five ſcore to the hundred; or twice that number of ſhillings, and once that number of groats, is the price of an hundred, at five ſcore and twelve to the hundred. Ex. gr. At three half pence the pound, the number of farthings is ſix. Therefore, twice ſix ſhillings, and once ſix pence (that is 12*s.* 6*d.*) is the price of an hundred, at five ſcore to the hundred. Again, twice ſix ſhillings, and once ſix groats (that is 14*s.*) is the price of an hundred, at five ſcore and twelve to the hundred. But of this enough, if not too much already.

CHAP. XII.

To divide a plot of Ground into any propoſed quantities. See Fig. 4.

SUppoſe A B C D E F G H I K L, a plot of ground, containing 54 acr. 2 roods, 28 poles, from the point A, I am required to ſhut off 18 acres, next the ſide B C. Draw A D, and meaſure the figure A B C D, which is 14 acr; 2 r. 3 p. that is 3 acr. 1. r. 37 p. or 557 poles too little. Draw again A F, and meaſure A D F, which is 1309. poles. Then by Chap. 5. Probl. 1. entring D F, the baſe of the triangle in 1309 on the lines of lines, and taking out 557, ſet it from

D to E, and draw A E. So have you the figure A B C D E 18 acres. Again, from the point G, I would ſet off 20 acres next the ſide A E, draw A G, and meaſure A E F G 15 acr. 0 r. 2 pol. whereof want 4 acr. 0 r. 38 pol. that 678 poles, then draw G K, and meaſure G A K 1113 poles. Laſtly, by Chap. 5. Probl. 1. enter A K, the baſe of the triangle in 1113 upon the lines of lines, and taking out 678, ſet it from A, and draw G L. So have you the figure A E F G L, twenty acres.

How ready the inſtrument would be for ſurveying with the help of a ſtaff, Ballſocket, and Needle, is obvious to any one that conſiders its graduated into 180 degrees.

CHAP. XIII.

So much of Geography as concerns finding the diſtance of any two places upon the Terreſtrial Globe.

HEre are three Caſes, and each of thoſe contains the ſame number of propoſitions.

CASE

CASE I.

When the two places differ in latitude only.

PROP. I.

When one place lies under the Equator, having no Latitude.

The latitude of the other place turned into miles, (reckoning 60 miles, the uſual compute, for a degree) is the diſtance ſought.

PROP. 2.

When both places have the ſame pole elevated, viz. North, or South.

Take the difference of their Latitudes, and reckoning 60 miles for a degree (as before) you have their diſtance.

PROP. 3.

When the two places have different poles elevated, viz. *one North, the other South.*

Adde two latitudes together, and that ſum turned into miles is the diſtance.

CASE 2.

When the two places differ in Longitude only.

PROP. 1.

When neither of them have any Latitude, but lie both under the Equator.

Their difference of Longitude turned into miles (as before) is their diſtance.

PROP. 2.

When the two places have the ſame Pole elevated.

The proportion is thus.

As the Radius is to the number 60, ſo is the coſine of the common latitude to the number of miles for one degree of longitude.

Multiply this number found by the difference of longitude, and that product is the diſtance in miles.

PROP. 3.

When the two places have different poles elevated.

As the Radius is to the coſine of the common latitude, ſo is the ſine of half the difference of longitude to the ſine of half the diſtance.

Wherefore this ſine of half the diſtance doubled and turned into miles, is the true diſtance.

CASE 3.

When the two places differ in Longitude and Latitude both.

PROP. 1.

When one of the places lies under the equator, having no Latitude.

As the radius is to the coſine of the difference of longitude, ſo is the coſine of the latitude to the coſine of the diſtance.

PROP. 2.

When both the places have the ſame pole elevated.

As the radius is to the coſine of the difference in longitude, ſo is the co-tangent of the leſſer altitude, to the tangent of a fourth ark.

Subtract this fourth ark out of the complement of the leſſer latitude, and keep the remain. Then,

As the coſine of the fourth ark is to the coſine of the remain, ſo is the ſine of the leſſer latitude to the coſine of the diſtance.

PROP. 3.

When the two places have different poles elevated.

As the radius is to the coſine of the difference in longitude, ſo is the co-tangent of either latitude to the tangent of a fourth ark.

Subtract the fourth out of the latitude not taken into the former proportion, and note the difference. Then,

As the coſine of the fourth ark is to the coſine of this difference, ſo is the ſine of the latitude firſt taken, to the coſine of the diſtance.

Chap. XIV.

Some uſes of the Inſtrument in Navigation, or plain Sailing.

HEre it will be neceſſary to premiſe the explication of ſome terms, and adjoyn two previous proportions.

1. The Compaſs being a circle, divided into 32 equal parts, called rumbs; one point or rumb is 11 d. 15 min. of a circle from the meridian: two points or rumbs is 22 d. 30 min. *&c.* of the reſt.

2. The angle which the needle, or point of the compaſs under the needle, makes with the meridian, or North and South line is called the courſe or rumb; but the angle which it makes with the Eaſt, and Weſt line, or any parallel, is named the complement of the courſe or rumb.

3. The departure is the longitude of that Port from which you ſet ſail.

4. The diſtance run, is the number of miles, or leagues (turned into degrees) that you have ſailed.

5. When you are in North latitude, and ſail North-ward, adde the difference of latitude to the latitude you ſailed from; and when you are in North latitude, and ſail Southward, ſubtract the difference of latitude from the latitude you ſailed, and you have the latitude you are in.

The ſame rule is to be obſerved in South latitude.

6. To finde how many miles anſwer to one degree of longitude in any latitude.

As the radius is to the number *60*, ſo is the coſine of the latitude to the number of miles for one degree.

7. To finde how many miles anſwer to one degree of latitude on any rumb.

As the coſine of the rumb from the Meridian, is to the number *60*, ſo is the radius to the number of miles.

The moſt material queſtions in Navigation are theſe four.

Firſt, To finde the courſe. Secondly, The diſtance run. Thirdly, The difference of latitude. Fourthly, The difference in longitude; and any two of theſe being given, the other two are readily found by the Square and Index.

Theſe two additional rulers were omitted in the firſt Chapter of this Treatiſe; ſith they are only for Navigation, and large Inſtruments of two, or three foot in length, which made me judge their deſcription moſt proper for this place: becauſe, ſuch as intend the Inſtrument for a pocket companion, will have no uſe of them.

The ſquare is a flat rule, having a piece, or plate faſtened to the head, that it may ſlide ſquare, or perpendicular to the outward ledge of the fixed piece. It hath the ſame line next either edge on the upper ſide, which is a line of equal parts, an hundred, wherein is equal to the radius of the degrees on the outward limb of the moveable piece.

The Index is a thin braſs rule on one ſide, having the ſame ſcale as the ſquare. On the other ſide is a double line of tangents, that next the left edge, being to a ſmaller; that next the right edge, to a larger radius.

For the uſe of theſe rulers, you muſt have a line of equal parts adjoyning to the line of ſines on the fixed piece, divided into 10 parts, ſtamped with figures, each of thoſe divided into 10 parts more; ſo that the whole line is divided into 100 parts, repreſenting degrees. Laſtly, let each of thoſe degrees be ſub-divided into as many parts, as the largeneſs of your ſcale will permit, for computing the minutes of a degree.

The Index is to move upon the pin on the fixed piece (where you hang the thread for taking altitudes) and that ſide of the Index (in any of the four former queſtions) muſt be upward, which hath the ſcale of equal parts.

The ſquare is to be ſlided along the outward ledge of the fixed piece. Then the general rules are theſe.

The difference of latitude is accounted on the line of equal parts, adjoyning to the ſines on the fixed piece.

The difference of longitude is numbred on the ſquare.

The diſtance run is reckoned upon the Index.

The courſe is computed upon the degrees on the limb from the head toward the end of the moveable piece.

But when any would work theſe Problems in proportions, let them note,

The diſtance run, difference of longitude, and difference of latitude, are all accounted on the line of numbers; the rumb or courſe is either a ſine or tangent.

This premiſed. I ſhall firſt ſhew how to reſolve any Problem by the ſquare and Index; and next adjoyn the proportions for the uſe of ſuch as have only pocket Inſtruments.

PROBL. I.

The courſe and diſtance run given, to finde the difference of latitude, and difference of longitude.

Apply the Index to the courſe reckoned on the limb from the head, and ſlide the ſquare along the outward ledge of the fixed piece, until the fiducial edge interſect the diſtance run on the fiducial edge of the Index. Then at the point of Interſection you have the difference of longitude upon the ſquare, and on the line of equal parts on the fixed piece, the ſquare ſhows the difference of latitude. The proportion is thus,

As the radius is to the diſtance run, ſo is the coſine of the courſe to the difference of latitude. Again,

As the radius is to the diſtance run, ſo is the ſine of the courſe to the difference in longitude.

PROBL. 2.

The courſe and difference of latitude given to find the diſtance run, and difference in longitude.

Slide the ſquare to the difference of latitude on the *line* of equal parts upon the fixed piece, and ſet the Index to the courſe on the limb. Then at the point of interſection of the ſquare and index, on the ſquare is the difference of longitude, on the index the diſtance run. The proportion is thus.

As the coſine of the courſe is to the difference of latitude, ſo is the radius to the diſtance run. Again,

As the radius is to the ſine of the courſe; ſo is the diſtance run to the difference of longitude.

PROBL. 3.

The courſe and difference in longitude given; to finde the diſtance run, and difference of latitude.

Apply the Index to the courſe on the limb, and the difference of longitude on the ſquare to the fiducial edge of the Index. Then at the point of interſection you have diſtance run on the index, and upon the line of equal parts on the fixed piece, the ſquare ſhows the difference of latitude. The proportion is thus.

As the ſine of the courſe is to the difference of longitude, ſo is the radius to the diſtance run. Again, As the radius is to the diſtance run, ſo is the coſine of the courſe to the difference of latitude.

PROBL. 4.

The diſtance run, and difference of latitude given to finde the courſe, and difference in longitude.

Slide the ſquare to the difference of latitude on the line of equal parts on the fixed piece, and move the index until the diſtance run numbred thereon, interſect the fiducial edge of the ſquare; then at the point of interſection you have the difference of longitude on the ſquare, and the fiducial edge of the Index on the limb ſhows the courſe.

As the diſtance run is to the difference of latitude, ſo is the radius to the coſine of the courſe. Again,

As the radius is to the diſtance run, ſo is the ſine of the courſe to the difference of longitude.

PROBL. 5.

The diſtance run, and difference of longitude given to finde the courſe, and difference of latitude.

Apply the diſtance run numbred on the fiducial edge of the index, to the difference of longitude, reckoned on the fiducial edge of the ſquare. Then on the line of equal parts upon the fixed piece, the ſquare ſhows the difference of latitude, and the index ſhows the courſe on the limb. The proportion is thus.

As the diſtance run is to the difference of longitude, ſo is the radius to the ſine of the courſe. Again,

As the radius is to the diſtance run, ſo is the coſine of the courſe, to the difference of latitude.

PROBL. 6.

The difference of latitude, and difference of longitude given, to finde the courſe, and diſtance run.

Apply the ſquare to the difference of latitude on the ſcale of equal parts upon the fixed piece, and move the index until its fiducial edge interſect the difference of longitude, reckoned on the ſquare. Then at the point of interſection you have the diſtance run upon the index; and the fiducial edge of the index upon the limb ſhows the courſe. The proportion is thus.

As the difference of latitude is to the difference of longitude, ſo is the radius to the tangent of the courſe. Again,

As the ſine of the courſe is to the radius, ſo is the difference of longitude to the diſtance run.

PROBL. 7.

Sailing by the Ark of a great Circle.

For this purpoſe the tangent lines on the index will be a ready help, uſing the leſſer for ſmall, and the greater tangent line, for great latitudes. The way is thus,

Account the pont 6c/o, on the outward limb of the moveable piece to be the point, or port of your departure; thereto lay the fiducial edge of the index, and reckoning the latitude of the Port you departed from upon the index, ſtrike a pin directly touching

ing it, into the table your inſtrument lies upon. This pin ſhall repreſent the Port of your departure. Therefore hanging a thread, or hair, on the center, whereon the index moves; and winding it about this pin. Count the difference of longitude 'twixt the port of your departure, and the Port you ſail toward, from *60|0* on the moveable piece toward 0/60, on the looſe piece; and thereto lay~~ing the ſame fiducial edge~~ of the *index*, reckon the latitude of this laſt Port upon the index, directly touching of it, ſtrike down another pin upon the table, and draw the thread ſtrait about this pin faſtening it thereto. This done, the thread betwixt the two pins repreſents the ark of your great Circle; and laying the fiducial edge of the index to any degree of difference of longitude accounted from *60/0* on the moveable piece, the thread ſhows upon the index what latitude you are in, and how much you have raiſed, or depreſſed the pole ſince your departure. On the contrary, laying the latitude your are in (numbred upon the index) to the thread, the index ſhows the difference of longitude upon the limb; counting from *60/0* on the moveable piece. So, that were it poſſible to ſail exactly by the ark of a great Circle, it would be no difficulty to determine the ſongitude in any latitude you make.

But I intend not a treatiſe of Navigation; wherefore let it ſuffice, that I have already ſhown how the moſt material Problems therein, may eaſily, ſpeedily, and (if the inſtrument be large) exactly, be performed by the inſtrument without the trouble of Calculation, or Projection.

CHAP. XV.

The Projection and Solution of the ſixteen Caſes in right angled Spherical Triangles by five Caſes. See Fig. 5.

THe fundamental Circle NBZC, is alwayes ſuppoſed ready drawn, and croſſed into quadrants, and the diameters produced beyond the Circle.

CASE I.

Given both the ſides ZD, and DR, to project the Triangle.

By a line of chords, prick off ZD, upon the limb, and draw the diameter DAE. Again, by a line of tangents, ſet half cotangent

tangent D R, upon A D, from A to R, then have you three points, *viz.* N R Z, to draw that ark, and make up the triangle. The center of which ark always lies on AC, (produced beyond C, if need requires) and is found by the interſection of the two arks made from R, and Z.

CASE 2.

Given one ſide Z D, and the hypothenuſe Z R, to project the Triangle.

Prick off Z D, and draw D A E, by Caſe 1. Again, ſet half the co-tangent Z R, on the line A Z, from A to F, and the tangent Z R, ſet from F to P, with the extent F P, upon the center P, draw the ark V F I, and where it interſects the diameter D A E, ſet R; then have you three points N R Z, to draw that ark, as in the former Caſe.

CASE 3.

Given the Hypothenuſe Z R, and the Angle D Z R, to project the Triangle.

Prick half the co-tangent D Z R, from A to S, and the ſecant D Z R, from S to T, upon the center T, with the extent T S, draw the ark N S Z. Again, by Caſe 2, draw the ark H F I, where theſe two arks interſect each other, ſet R. Laſtly, lay a ruler to A R, and draw the diameter DRAE, and your triangle is made.

CASE 4.

Given one ſide Z D, and its adjacent Angle D Z R, to project the triangle.

Prick off Z D, and draw D A E, by Caſe 1. Again, by Caſe 3, draw the ark N S Z, where this ark interſects the diameter DAE, ſet R, and your triangle is made. Or,

Given the ſide D R, and its oppoſite angle D Z R, you may project the triangle.

Draw the ark N S Z, as before. Again, take half the co-tangent D R, and with that extent upon A, the center, croſs the ark N S Z, ſetting R, at the point of interſection. Laſtly, lay a ruler to A R, and draw the diameter D R A E, which makes up your triangle.

The triangle projected in any of the four former Caſes, to meaſure any of the ſides, or angles, do thus.

Firſt, the ſide Z D, is found by applying it to a line of chords. Secondly, R A, applyed to a line of tangents, is half the co-tangent D R. Thirdly, A S, applyed to a line of tangents is the co-tangent D Z R. Fourthly, ſet half the tangent D Z R, from A to L, then is L the pole point, and laying a ruler to L R, it cuts the limb at V, and the ark Z V, upon the line of chords, gives the hypothenuſe Z R. Fifthly, prick Z D from C to K, a ruler laid from R to L, cuts the limb at G, then G K, upon a line of chords, is the quantity of Z R D.

CASE. 5.

The two oblique angles, D Z R, and Z R D given, to preſent the Triangle. See Fig. 6.

This is no more than turning the former triangle. Thus,

Draw the ark N S Z, by Caſe 1, and ſet half the tangent of that ark from A to L. Again, ſet half the co-tangent D R Z, from A to F, and the ſecant of D R Z, from F to L, upon the center A, with the extent A P. Draw the ark P G, and with the extent F P, from L, croſs the ark P G in G. Laſtly, upon the Center G, with the extent G L. Draw the ark R D F L, and your triangle is made.

The triangle projected you may meaſure off the ſides and hypothenuſe. Thus,

Firſt, the hypothenuſe Z R, is meaſured by a line a line of chords. Secondly, a ruler laid to L D, cuts the limb at H, and Z H, upon a line of chords, is the meaſure of the ark Z D. Thirdly, draw A G, and ſet half the tangent D R Z, from A to V, apply a ruler to V D, it cuts the limb at E, then R E, upon a line of chords, meaſure the ark R D.

Note. The radius to all the chords, tangents, and ſecants, uſed in the projection, and meaſuring, any ark or angle, is the ſemidiameter of the fundamental circle.

CHAP. XVI.

The projection and ſolution of the 12 Caſes in oblique angled ſpherical triangles in ſix Caſes. See Fig. 7.

THe fundamental circle N H Z M, is alwayes ſuppoſed ready drawn, and croſſed into Quadrants, and the Diameters produced beyond the Circle.

CASE I.

The three ſides, Z P, P Z, and Z S, given, to project the Triangle.

By a line of chords prick off Z P, and

draw the diameter P C T, croſſing it at right angles in the center with ÆCE, ſet half the co-tangent PS, from C to G, and the ſecant P S from C to R; upon the center R, with the extent R G, draw the the ark FGL. Again, ſet half the co-tangent Z S, from C to D, and the tangent Z S, from D to O, with the extent O D, upon the center O, draw the ark B D P, mark where theſe two arks interſect each other as at S. Then have you three points T S P, to draw that ark, and the three points N S P, to draw that ark, which make up your triangle.

CASE 2.

Given two ſides Z S, and Z P, with the comprehended Angle P Z S, to project the Triangle.

Prick off Z P, and draw PCT, and ÆCE, and the ark B D P, by Caſe 1. Again, ſet the tangent of half the exceſs of the angle P Z S above 90, from C to W, and the co-ſecant of that exceſs from W to K, upon the center K, with the extent K W; draw the ark N W Z, which cuts the ark B D P in S. Then have you the three points T S P, to draw that ark which makes up the triangle.

CASE

CASE 3.

Two Angles S Z P, and ZPS, with the comprehended ſide Z P, given, to project the Triangle.

Prick off Z P, and draw the lines P C T, and Æ C E, by Caſe 1, and the angle NWZ by Caſe 2. Laſtly, ſet half the co-tangent ZPS from C to X, and the ſecant Z P S, from X to V, upon the center V, with the extent V X, draw the ark T X S P, and the triangle is made.

CASE 4.

Two ſides, ZP, and PS, with the Angle oppoſite to one of them SZP, given, to project the Triangle.

Prick off ZP, and draw PCT, and ÆCE, by Caſe 1. and the angle SZP by Caſe 2. Laſtly, by Caſe 1. draw the ark FGL, and mark where it interſects NWZ, as at S, then have you the three points TSP, to draw that ark, and make up the triangle.

CASE 5.

Two Angles SZP, and ZPS, with the ſide oppoſite to one of them ZS, given, to project the Triangle.

Draw the ark BDP, by Caſe 1. and the ark NWZ, by Caſe 2. at the interſection of theſe two arks, ſet S, with the tangent of the angle ZPS, upon the center C. ſweep the ark V△I. Again, with the ſecant of the ark ZPS upon the center S, croſs the ark V△I, as at the points V and I. Then in caſe the hypothenuſe is leſs than a quadrant (as here) the point V, is the center, and with the extent VS, draw the ark TSP, which makes up the triangle. But in caſe the hypothenuſe is equal to a quadrant, △, is the center; if more than a quadrant, I, is the center; in which caſes the extent from △, or I, to S, is the ſemidiameter of the ark TSP.

CASE 6.

Three Angles ZPS, and PZS, and ZSP, given, to project the Triangle. See Fig. 7. and 8.

The angles of any ſpherical triangle may be converted into their oppoſite ſides, by taking the complement of the greateſt angle to a Semicircle for the hypothenuſe, or greateſt ſide. Wherefore by Caſe 1. make the ſide ZP, in Fig. 4. equal to the angle ZSP, in Fig. 3, and the ſide ZS, in Fig. 4. equal to the angle ZPS, in Fig. 3. and the ſide PS, in Fig. 4. equal to the complement of the angle PZS, to a Semicircle in Fig. 3. Then is your triangle projected where the angle ZPS in Fig. 4. is the ſide ZS, Fig. 3. Again, the angle ZSP, Fig. 4. is the ſide ZP, in Fig. 3. Laſtly, the complement of the angle PZS to a Semicircle in Fig. 4. is the meaſure of the hypothenuſe, or ſide P S, in Fig. 3.

The Triangle being in any of the former Caſes projected, the quantity of any ſide or angle may be meaſured by the following rules.

Firſt, The ſide Z P, is found by applying it to a line of chords. Secondly, CX, applyed to a line of tangents, is half the co-tangent of the angle ZPS. Thirdly, C W applyed to a line of tangents, is half the co-tangent of the exceſs of the angle SZP, above 90. Fourthly, ſet half the tangent of the angle ZPS, from C, to п, a ruler laid to п S, cuts the limb at F; then P F, applyed to a line of chords, gives the ſide PS. Fifthly, take the complement of the angle PZS, to a Semicircle, and ſet half the tangent of that complement from C, to λ, a ruler laid to λS, cuts the limb at B, and ZB, applyed to a line of chords, gives the ſide ZS. Sixthly, a ruler laid to Sλ, cuts the limb at L. Again, a ruler laid to Sπ, cuts the limb at Φ, and L Φ, applyed to a line of chords, gives the angle ZSP.

The end of the firſt Book.

An Appendix to the firſt Book.

THe ſights which are neceſſary for taking any Altitude, Angle, or diſtance (without the help of Thread or Plummet) are only three, *viz.* one turning ſight, and two other ſights, contrived with chops, ſo that they may ſlide by the inward or outward graduated limbs. The turning ſight hath only two places, either the center at the head, or the center at the beginning of the line of ſines on the fixed piece; to either of which (as occaſion requires) its faſtened with a ſorne. The center at the head ſerving for the graduations next the inward limb of the looſe piece. And the center at the beginning of the line of ſines ſerving for all the graduations next the outward limb of the moveable and looſe piece both. Yet becauſe it is requiſite to have pins to keep the looſe piece cloſe in its place. You may have two ſights more to ſupply their place (which ſometimes you may make uſe of) and ſo the number of ſights may be five, *viz.* two ſliding ſights, one turning ſight, and two pin ſights, to put into the holes at the end of the fixed and moveable piece, to hold the tenons of the looſe piece cloſe joynted. Every one of theſe ſights hath a fiducial (or perpendicular) line, drawn down the middle of them, from the top to the bottom, where this line toucheth the graduations on the limb, is the point of obſervation.

The places of theſe ſights have an oval proportion, about the middle of them, only leaving a ſmall bar of braſs, to conduct the fiducial line down the oval cavity, and ſupport a little braſs knot (with a ſight hole in it) in the middle of that bar, which is ever the point to be looked at. There are two wayes of obſerving an altitude with help of theſe ſights. The one when we turn our face toward the object. This is called a forward obſervation in which you muſt alwayes ſet the turning ſight next your eye. This way of obſervation will not exactly give an altitude above 45 degrees.

The other way of obſerving an altitude is peculiar to the Sun in a bright day, when we turn our back toward the Sun. This is termed a backward obſervation; wherein

You muſt have one of the ſliding ſights next Your eye, and the turning ſight toward the Horizon. This ſerves to take the Suns altitude without thread, or plummet, when it is near the Zenith.

PROBL. I.

To finde the Suns altitude by a forward obſervation.

Serve the turning ſight to the center of thoſe graduations you pleaſe to make uſe of (whether on the inward or outward limb) and place the two ſliding ſights upon the reſpective limb to that center; this done, look by the knot of the turning ſight (moving the inſtrument upward or downward) until you ſee the knot of one of the ſliding ſights directly againſt the Sun, then move the other ſliding ſight, until the knot of the turning ſight, and the knot of this other ſliding ſight be againſt the horizon; then the degrees intercepted 'twixt the fiducial lines of the ſliding ſights on the limb, ſhew the altitude required.

PROBL. 2.

To finde the diſtance of any two Stars, &c. by a forward obſervation.

Serve the turning ſight to either center, and apply the two ſliding ſights to the reſpective limb (holding the inſtrument with the proportional ſide downward) and applying the turning ſight to your eye, ſo move the two ſliding ſights either nearer together, or further aſunder, that you may by the knot of the turning ſight ſee both objects even with the knots of their reſpective ſliding ſights, then will the degrees intercepted 'twixt the fiducial lines of the object ſights on the limb ſhow the true diſtance. By this means you may take any angle for ſurveying, *&c.*

PROBL. 3.

To finde the Suns Altitude by a backward Obſervation.

Serve the turning ſight to the center at the beginning of the line of ſines, and apply one of the ſliding ſights to the outward limb of the looſe piece, and the other to the outward limb of the moveable piece; and turning your back toward the Sun, ſet the ſliding ſight upon the moveable piece next your eye; and ſlide it upward or downward toward the end, or head, until you ſee the ſhadow of the little bar, or edge, of the ſight on the looſe piece fall directly on the little bar on the turning ſight; and at the ſame time the bar of the ſight next your eye, and the bar of the turning ſight to be in a direct line with the Horizon. Then will the degrees on the limb intercepted 'twixt the fiducial lines of the ſliding ſights (if you took the ſhadow of the bar) or 'twixt the fiducial line of the ſliding ſight next your eye, and the edge of the other ſliding ſight (when you took the ſhadow of the edge) be the true altitude required.

ΣΚΙΟΓΡΑΦΙΑ.

OR,

The Art of Dyalling for any plain Superficies.

Lib. II.

Chap. I.

The diſtinction of Plains, with Rules for knowing of them.

ALL plain Superficies are either horizontal, or ſuch as make Angles with the Horizon.

Horizontal plains are thoſe, that lie upon an exact level, or flat.

Plains, that make Angles with the Horizon are of three ſorts.

1. Such as make right angles with the Horizon, generally known by the name of erect, or upright plains.

2. Such as make acute angles with the horizon, or have their upper edge leaning toward you, uſually termed inclining plains.

3. Such as make obtuſe angles with the horizon, or have their upper edge falling from you, commonly called reclining plains.

All theſe three ſorts are either direct, *viz.* Eaſt, Weſt, North, South. Or elſe

Declining {
From South, toward Eaſt, or Weſt.
From North, toward Eaſt, or Weſt.

All plain Superficies whatſoever are comprized under one of theſe terms. But before we treat of the affections, or delineation of Dials for them; it will be requiſite to acquaint you with the nature of any plain, which may be found by the following Problems.

PROBL. I.

To finde the reclination of any Plain.

Apply the outward ledge of the moveable piece to the Plain with the head upward,

ward, and reckoning what number of degrees the thread cuts on the limb (beginning your account at 30. on the looſe piece, and continuing it toward 60|0 on the moveable piece) you have the angle of reclination. If the thread falls directly on 60|0 upon the moveable piece, its an horizontal; if on 30. on the looſe piece, its an erect plain.

PROBL. 2.

To finde the inclination of any Plain.

Apply the outward ledge of the fixed piece to the plain, with the head upward, and what number of degrees the thread cuts upon the limb of the looſe piece, is the complement of the plains inclination.

PROBL. 3.

To draw an Horizontal Line upon any Plain.

Apply the proportional ſide of the Inſtrument to the plain, and move the ends of the fixed piece upward, or downward, until the thread falls directly on 60|0. upon the looſe piece; then drawing a line by the outward ledge of the fixed piece, its horizontal, or paralel to the horizon.

PROBL. 4.

To draw a perpendicular Line upon any Plain.

When the Sun ſhineth hold up a thread with a plummet againſt the plain, and make two points at any diſtance in the ſhadow of the thread upon the plain, lay a ruler to theſe points, and the line you draw is a perpendicular.

PROBL. 5.

To finde the declination of any Plain.

Apply the outward ledge of the fixed piece to the horizontal line of your plain, holding your inſtrument paralel to the horizon. This done, lift up the thread and plummet, until the ſhadow of the thread fall directly upon the pin hole on the fixed piece (where you hang the thred to take altitudes) Then obſerve how many degrees the ſhadow of the thread cuts in the limb, either from the right hand, or from the left hand o|60. upon the looſe piece; and immediately taking the altitude of the Sun. By lib. 1. cap. 2. Probl. 9, 10, 11. finde the Suns Azimuth from South. And,

When you make this obſervation in the morning, theſe Caſes determine the declination of the plain.

CASE 1.

When the ſhadow of the thread upon the limb falls on the right hand 0/60 on the looſe piece, take the difference of the ſhadow, and Azimuth (by ſubtracting the leſſe out of the greater) and the reſidue or remain is the plains declination. From South toward Eaſt, when the Azimuth is greater than the ſhadow. From South toward Weſt, when the ſhadow is greater than the azimuth, when the ſhadow and azimuth are equal, its a direct South plain. When the difference is juſt 90. its a direct Eaſt, when above 90. ſubtract the difference from 180, and the remain is the declination from North toward Eaſt.

CASE 2.

When the ſhadow falls on the left hand 0/60. Adde the azimuth and ſhadow together,

ther, that ſum is the plains declination; from South toward Eaſt, when under *90*; if it be juſt *90*, its a direct Eaſt. If above *90*. ſubtract it from 180. the remain is the declination from North toward Eaſt. When the ſum is above 180. ſubtract 180 from it, and the remain is the declination from North toward Weſt.

CASE 3.

When the ſhadow falls upon o|*60*. the azimuth is the plains declination. When under *90*, its South-Eaſt, when equal to *90*. direct Eaſt, when above *90*. ſubtract it from 180. the remain is the declination from North toward Eaſt.

If you make the obſervation afternoon, the following Caſes will reſolve you.

CASE 4.

When the ſhadow falls on the left hand o *60*. the difference 'twixt the ſhadow and azimuth is the declination; when the ſhadow is more than the azimuth it declines South-Eaſt, when leſs, South-Weſt. When the ſhadow and azimuth are equal, its a direct South plain, when their difference is equal to *90*.

its a direct Weſt; when the difference exceeds 90. ſubtract it from 180. the remain is the declination North-Weſt.

CASE 5.

When the ſhadow falls on the right hand 0/60. take the ſum of the ſhadow and azimuth, and that is the declination from South toward Weſt, when under 90. when juſt 90. its a direct Weſt plain; when more than 90. ſubtract it from 180. the remain is the declination North-Weſt; when the ſum is above 180. ſubtract 180 from it, and the remain is the declination from North toward Eaſt.

CASE 6.

When the ſhadow falls upon 0/60. the azimuth is the quantity of declination. From South toward Weſt, when under 90. when equal to 90. its direct Weſt; when more than 90. ſubtract it from 180. The remain is the declination from North toward Weſt.

PROBL. 6.

To draw a Meridional Line upon a Horizontal Plain.

Draw firſt a circle upon the plain, and holding up a thread and plummet (when the Sun ſhines) ſo that the ſhadow of the thread may paſs through the center of the circle, make a point in the circumference where the ſhadow interſects it. At the ſame time finding the Suns Azimuth from South, by a line of chords, ſet it upon the limb of the circle from the interſection of the ſhadow toward the South, and it gives the true South point. Wherefore laying a ruler to this laſt point and the center, the line you draw is a true Meridian.

CHAP. II.

The affections of all ſorts of Plains.

Sect. 1.

The affections of an horizontal Plain.

1. THe ſtyle, or cock of every horizontal Dial, is an angle equal to that latitude for which the Dial is made.

2. The place of the ſtyle is directly upon the meridional, or twelve a clock line, and the angular point muſt ſtand in the center of the hour lines.

3. The rule for drawing the hour lines before ſix in the morning, is to draw the reſpective hour lines afternoon beyond the center; or for the hour lines after ſix at evening, draw the reſpective hour lines in the morning beyond the center.

4. To place an horizontal Dial upon the plain; firſt draw a Meridian line upon the plain by Cap. 1. Probl. 6. and lay in the line of 12. exactly thereon, with the angular point of the ſtyle toward the South, faſten the Dial upon the plain.

Sect

Sect. 2.

The affections of erect, direct South and North Plains.

1. The ſtyle in both theſe is an angle equal to the complement of the latitude for which the Dial is made to ſtand upon the Meridian line, or perpendicular of your plain, with the angular point in the center of the hour lines, and that point in South alwayes upward, in North alwayes downward.

2. To prick off the Dial from your paper draught upon the plain, lay the hour line of *6.* and *6.* upon the plains horizontal; and applying a ruler to the center, and each hour line, tranſmit the hour lines from your paper draught to the plain.

Sect. 3.

The affections of erect, direct East and West Plains.

1. In both theſe the ſtyle may be a pin or plate, equal in length or heighth to the radius of the tangents, by which you draw the Dial.

2. The ſtyle in both of them is to ſtand directly upon the hour of ſix, and perpendicular to the plain.

Sect. 4.

The affections of erect declining Plains.

1. Theſe are of two ſorts, either ſuch as admit of centers to the hour lines and ſtyle, or ſuch as cannot with conveniency (becauſe of the lowneſs of the ſtyle, and nearneſs of the hour lines to each other) be drawn with a center to thoſe lines. Of this latter ſort are all ſuch plains, whoſe ſtyle is an angle leſs than 15 degrees. For where the angle of the ſtyle is more than 15 degrees; thoſe Dials may be drawn with a center to the hour lines.

2. In all erect declining plains with centers the Meridian is the plains perpendicular. In thoſe that admit not of centers in their delineation, the meridian is parallel to the plains perpendicular.

3. In all declining plains the ſubſtile, or line whereon the ſtyle is to ſtand, muſt be placed on that ſide the meridian, which is contrary to the coaſt of declination; and alſo in ſuch decliners as admit of centers, the

angle 'twixt 12. and *6.* is to be ſet to the contrary coaſt to that of declination.

4. In all decliners, without centers, the inclination of meridians is to be ſet from the ſubſtyle toward the coaſt of declination.

5. The proportions in all erect decliners, for finding the height of the ſtyle, the diſtance of the ſubſtyle from the meridian, the angle of twelve and ſix, with the inclination of meridians (all which may be wrought either by the canon, or exactly enough for this purpoſe by the inſtrument) are as followeth.

To finde the Styles height above the Subſtile.

As the radius is to the coſine of the latitude, ſo is the coſine of declination, to the ſine of the ſtyles height.

To finde the Subſtyles diſtance from the Meridian.

As the radius is to the ſine of declination, ſo is the co-tangent of the latitude to the tangent of the ſubſtyle from the Meridian.

To finde the angle of twelve and ſix.

As the co-tangent of the latitude is to the radius, ſo is the ſine of declination, to the co-tangent of ſix from twelve.

To finde the inclination of Meridians.

As the ſine of the latitude is to the radius, ſo is the tangent of declination to the tangent of inclination of Meridians.

6. All North decliners with centers have the angular point of the ſtyle downward, and all South have it upward.

7. All North decliners without centers have the narroweſt end of the ſtyle downward, all South have it upward.

8. In all decliners without centers take ſo much of the ſtyle as you think convenient, but make points at its beginning and end upon the ſubſtyle of your paper draught, and tranſmit thoſe points to the ſubſtyle of your plain, for direction in placing your ſtyle thereon.

9. In all North decliners the Meridian, or inclination of Meridians is the hour line of twelve at mid-night: in South decliners, at noon, or mid-day. This may tell you the true names of the hour lines.

10. In tranſmitting theſe Dials from your paper draught to your plain, lay the horizontal of your paper draught, upon the horizontal line of the plain, and prick off the hours and ſubſtyle.

Sect. 5.

The affections of direct $\left.\begin{matrix}\text{re}\\\text{in}\end{matrix}\right\}$*clining Plains.*

For South Recliners, and North Incliners.

1. The difference 'twixt your co-latitude, and the $\left.\begin{matrix}\text{re}\\\text{in}\end{matrix}\right\}$clination is the elevation, or height of the ſtyle.

2. When the $\left.\begin{matrix}\text{re}\\\text{in}\end{matrix}\right\}$clination exceeds your co-latitude, the contrary pole is elevated ſo much as the exceſs. Ex. gr. a North recliner, or South incliner 50. d. in lat. 52. 30. min. the exceſs of the $\left.\begin{matrix}\text{re}\\\text{in}\end{matrix}\right\}$clination, to your co-latitude is 12. d. 30. min. and ſo much the North is elevated on the recliner, and the South pole on the incliner.

3. When the $\left.\begin{matrix}\text{re}\\\text{in}\end{matrix}\right\}$clination is equal to the co-latitude, its a polar plain.

For South incliners, and North recliners.

1. The Sum of your co-latitude, and the $\left.\begin{matrix}\text{re}\\\text{in}\end{matrix}\right\}$clination is the ſtyles elevation.

2. When the $\left.\begin{matrix}\text{re}\\\text{in}\end{matrix}\right\}$clination is equal to your latitude, its an equinoctial plain, and the Dial is no more than a circle divided into 24. equal parts, having a wyer of any convenient length placed in the center perpendicular to the plain for the ſtyle.

3. When the $\left.\begin{matrix}\text{re}\\\text{in}\end{matrix}\right\}$clination is greater than your latitude, take the ſumme of the $\left.\begin{matrix}\text{re}\\\text{in}\end{matrix}\right\}$clination of your co-latitude from 180. and the reſidue, or remain is the ſtyles height. But in this caſe the ſtyle muſt be ſet upon the plain, as if the contrary pole was elevated, *viz.* Theſe North recliners muſt have the center of the ſtyle upward, and the South incliners have it downward.

Note. In all $\left.\begin{array}{c}\text{South}\\\text{North}\end{array}\right\}$re-in-cliners, for their delineation the ſtyles height is to be called the co-latitude, and then you may draw them as erect direct plains, for South, or North (as the former rules ſhall give them) in that latitude, which is the complement of the ſtyles height.

For direct Eaſt and Weſt $\left.\begin{array}{c}\textit{re}\\\textit{in}\end{array}\right\}$*cliners.*

1. In all Eaſt and Weſt $\left.\begin{array}{c}\text{re}\\\text{in}\end{array}\right\}$cliners, you may refer them to a new latitude, and new declination, and then deſcribe them as erect declining plains.

2. Their new latitude is the complement of that latitude where the plain ſtands, and their new declination is the complement of their $\left.\begin{array}{c}\text{re}\\\text{in}\end{array}\right\}$clination: But to know which way you are to account this new declination, remember all Eaſt and Weſt recliners are North-Eaſt, and North-Weſt decliners. All Eaſt and Weſt incliners are South-Eaſt, and South-Weſt decliners.

3. Their new latitude and declination known, you may by Sect 4. par. 5. finde the

ſubſtyle from the Meridian, height of the ſtyle, angle of twelve and ſix, and inclination of Meridians, uſing in thoſe proportions the new latitude and new declination inſtead of the old.

4. In all Eaſt and Weſt $\left.\begin{array}{c}\text{re}\\\text{in}\end{array}\right\}$cliners with centers, the Meridian lies in the horizontal line of the plain; in ſuch as have not centers, its paralel to the horizontal line.

Sect. 6.

The affections of declining $\left.\begin{array}{c}\text{re}\\\text{in}\end{array}\right\}$*clining Plains.*

The readieſt way for theſe, is to refer them to a new latitude, and new declination, by the ſubſequent proportions.

1. *To finde the new Latitude.*

As the radius is to the coſine of the plains declination, ſo is the co-tangent of the $\left.\begin{array}{c}\text{re}\\\text{in}\end{array}\right\}$clination to the tangent of a fourth ark.

In $\left\{\begin{array}{l}\text{South recliners}\\\text{North incliners}\end{array}\right\}$ get the difference 'twixt this fourth ark, and the latitude of your place, and the complement of that difference is the new latitude ſought. If the fourth ark be leſs than your old latitude, the contrary pole is elevated ; if equal to your old latitude, its a polar plain.

In $\left\{\begin{matrix}\text{South incliners}\\ \text{North recliners}\end{matrix}\right\}$ the difference 'twixt the fourth ark, and the complement of your old latitude is the new latitude. If the fourth ark be equal to your old co-latitude, they are equinoctial plains.

2. *To finde the new declination.*

As the radius is to the coſine of the $\left.\begin{matrix}\text{re}\\ \text{in}\end{matrix}\right\}$clination , ſo is the ſine of the old declination to the ſine of the new.

3. *To finde the angle of the Meridian with the Horizontal Line of the Plain.*

As the radius is to the tangent of the old declination, ſo is the ſine of $\left.\begin{matrix}\text{re}\\ \text{in}\end{matrix}\right\}$ clination , to the co-tangent of the angle of the meridian with the horizontal line of the plain.

This

This gives the angle for its ſcituation. Obſerve, in

North incliners South recliners	leſs than a polar, the Meridian lyes.	above	That end of the Horizontall Line contrary to the Coaſt of Declination.
		below	
	more than a polar, the Meridian lyes.	below	That end of the Horizontall next the Coaſt of Declination.
		above	

- North recliners
- South incliners
  - leſs than an equinoctial, the Meridian lyes
    - above
    - below
      - That end of the Horizontal next the Coaſt of Declination.
      - In North recliners this Meridian is 12. at midnight.
  - equal to an equinoctial the Meridian deſcends below the Horizontal at that end contrary to the coaſt of Declination, and the ſubſtyle lies in the hour line of ſix.
  - more than an equinoctial, the Meridian lyes.
    - below
    - above
      - That end of the horizontal contrary to the declination.
      - In South incliners this Meridian is only uſeful for drawing the Dial, and placing the ſubſtyle, for the hour lines muſt be drawn through the center to the lower ſide.

After you have by the former proportions and rules found the new latitude, new declination, the angle and ſcituation of the meridian, your firſt buſineſs in delineating of the Dial will be (both for ſuch as have centers, and ſuch as admit not of *centers*) to ſet off the meridian in its proper coaſt and quantity. This done, by Sect. 4. Paragr. 5. of this Chapter, finde the ſubſtyles diſtance from the Meridian, the height of the ſtyle, angle of twelve and ſix (and for Dials without centers, the inclination of Meridians) in all thoſe proportions, uſing your new latitude and new declination, inſtead of the old, and ſetting them off from the Meridian, according to the directions in Paragr. 3. and 4. you may draw the Dials by the following rules, for erect declining plains. In placing of them, lay the horizontal line of your paper draught upon the horizontal line of your plain, and prick off the ſubſtyle and hour lines.

Only obſerve. That ſuch South-Eaſt, or South-Weſt recliners, as have the contrary pole elevated muſt be deſcribed as North-Eaſt, and North-Weſt decliners, and ſuch North-Eaſt, and North-Weſt incliners as have the contrary pole elevated, muſt be deſcribed as South-Eaſt, and South-Weſt decliners, which will direct you which way to ſet off the ſubſtyle, and hour line of ſix from the Meridian in thoſe oblique plains, which admit of centers, or the ſubſtyle from the Meridian, and the inclination of Meridians from the ſubſtyle in ſuch as admit not of Centers.

4. Declining polar plains muſt have a peculiar calculation for the ſubſtyle and inclination of Meridians, which is thus.

To finde the Angle of the Subſtile with the Horizontal Line of the Plain.

As the radius is to the ſine of the polar plains reclination, ſo is the tangent of declination to the co-tangent of the ſubſtyles diſtance from the horizontal line of the plain.

To finde the inclination of Meridians.

As the radius is to the ſine of the latitude, ſo is the tangent of declination to the tangent of inclination of Meridians.

5. The reclining declining polar hath the ſubſtyle lying below that end of the horizontal line that is contrary to the coaſt of declination. The inclining declining polar

hath the ſubſtyle lying above that end of the horizontal line, contrary to the coaſt of declination.

Chap. III.

The delineation of Dials for any plain Superficies.

HEre it will be neceſſary to premiſe the explication of ſome few terms and ſymbols, which for brevity ſake we ſhall hereafter make uſe of. Ex. gr.

Rad. denotes the radius, or ſine *90.* or tangent 45.

Tang. is the tangent of any arch or number affixed to it.

Cos. is the coſine of any arch or number of degrees, or what it wants of *90.* Ex. gr. cos. 1*9.* is what 1*9.* wants of *90.* that is 71.

Co-tang. is the co-tangent of any ark or number affixed to it, or what it wants of *90.* Ex. gr. co-tangent 30. is what 30. wants of *90.* that is 60.

--This is a note of equality in lines, numbers, or degrees. Ex. gr. AB--CD. That is, the line AB. is equal unto, or of the ſame length as the line CD. Again, ABC--EFG. that is the angle ABC. is of the ſame quantity, or number of degrees, as the angle EFG. Once more AB--CD--FG--tang. 15. That is AB. and CD. and FG. are all of the ſame length,and that length is the tangent of 15.d.

= This is a note of two lines being paralel unto, or equidiſtant from each other. Ex. gr. FL = RS. that is the line FL. is paralel unto, or equi-diſtant from the line RS.

Sect. 1.

To delineate an horizontal Dial. See Fig. 9.

First draw the ſquare BCDE. of what quantity the plain will permit. Then make

AF--AG--HD--HE--ſine of the latitude, and AH--Radius. Enter HD. in tang. 45. and keeping the Sector at that gage, ſet off HI--HK--tang. 15. and HO--HL--tang. 30. Again, enter FD. in tang. 45. and ſet off FQ--GN--tang. 15. and FP--GM--tang. 30. This done,

Draw AQ. AP. AD. AO. AI. for the hour lines of 5. 4. 3. 2. 1. afternoon. Again,

Draw AK. AL. AE. AM. AN. for the hour lines of 11. 10. 9. 8. 7. before noon.

The line FAG. is for ſix and ſix. In the ſame manner you may prick the quarters of an hour, reckoning three tangents, and 45. mi-

minutes, for a quarter. How to draw the hour lines before, and after ſix, was mentioned, Chap. 2. Sect. 1.

Sect. 2.

To deſcribe an erect, direct South Dial. See Fig. 10.

Draw ABCD. a rect-angle parallelogram. Then make AE--EB--CF--FD--cos. of your latitude. And EF--AC--BD--ſine of your latitude. Enter CF. in tang. 45. and lay down FK--FL--tang. 15. and FI--FM--tang. 30. Again, enter AC. in tang. 45. and lay down AG--BO--15. and AH--BN--tang. 30. with a ruler draw the lines EG. EH. EC. EF. EK. for the hour lines of 7. 8. 9. 10. 11. in the morning. and EO. EN. ED. EM. EL. for 5. 4. 3. 2. 1. afternoon, the line AEB. is for ſix and ſix. The line EF. for twelve.

The deſcription of a direct North-Dial differs nothing from this, only the hour lines from Sun riſe to ſix in the morning, and from ſix in the evening, until Sun ſet; muſt be placed thereon, by drawing the reſpective morning and evening hours beyond the center as in the horizontal. See Fig. 11.

Sect. 3.

To deſcribe an erect, direct Eaſt Dial. See Fig. 12.

Having drawn ABCD. a rectangle parallellogram, fix upon any point in the lines AB. and CD. for the line of ſix, provided the diſtance from that point to A. being entred radius in the line of tangents, the diſtance from thence to B. may not exceed, nor much come ſhort of the tangent 75. This point being found, enter the diſtance from thence to A. (which we ſhall call *6.* A.) radius in the line of tangents, and keeping the ſector at that gage, lay down upon the lines AB. and CD. *6,* 11.--tang. 75. *6.* 10.--tang. 60. and *6.* 9.--*6.* A.--tang. 45. and *6.* 8.--*6.*--4.--tang. 30. Laſtly, *6.* 7.- *6.* 5.--tang. 15. draw lines from theſe points on AB. to the reſpective points on CD. and you have the hours.

To place it on the plain, draw the angle DCE.--co-latitude, and laying ED. on the horizontal line of the plain, prick off the hours.

The ſame rules ſerve for delineation of a Weſt Dial, only as this hath morning, that muſt be marked with afternoon hours.

Sect.

Sect. 4.

To deſcribe an erect declining Dial, having a Center. See Fig. 14.

Draw the ſquare BCDE, and make AC--AK of quantity what you pleaſe. Again, draw AG. 12. = CE. and KHF. = AG. 12. By a line of chords, ſet off the angles of the ſubſtyle, ſtyle, and hour of ſix from twelve; having firſt found theſe angles by Chap. 2. Sect. 4. Paragr. 5. This done; make a mark in A 6. where it interſects KF. as at H. Then enter A K. in the ſecant of the plains declination, and keeping the Sector at that gage, take out the ſecant of the latitude, which place from A to G. upon the the line A. 12. and again, from H. (which is the interſection of the paralel FK. with the line of ſix) unto F. This done, lay a ruler to the points F. and G. and draw a line until it interſects CE. as FG. 3. Laſtly,

Enter GF. in tang. 45. and ſet off GL--GN--tang. 15. and GM--GO--tang. 30. Again, enter HF. in tang. 45. and ſet off HR--tang. 15. and HP--tang. 30. A ruler laid to theſe points, and the center, you may draw the hours lines from ſix in the morning unto three afternoon. For the other hour lines, do thus, Produce the line EC. and likewiſe HA. beyond the center, until they interſect each other as at S. Then ſetting off ST--HR. and S. 4.--HP. you have the hour points after three in the afternoon, until ſix, although none are proper beyond the hour line of four; only by drawing them on the other ſide the center, they help you to the hour lines before ſix in the morning.

Sect. 5.

To deſcribe an erect declining Plain without a Center. See Fig. 12.

The delineation of theſe Dials is the moſt difficult of any, and therefore I ſhall be the larger in their deſcription.

1. By Chap. 2. S. 4. Paragr. 5. finde the angles of the ſtyle, ſubſtyle, and inclination of Meridians.

2. Having found the inclination of Meridians, make the following Table for the diſtance of every hour line and quarter from the ſubſtyle. Where I take for example a North plain declining Eaſt 72. d. 45. min. in lat. 52. 30. min.

The manner of drawing this Table is thus. The inclination of Meridians (which becauſe its a North decliner, is twelve at midnight.) I finde 76. d. 30. min. Now conſidering the Sun never riſeth till more than half an hour after three in this latitude, I know that one quarter before four is the firſt line proper for this plain: Therefore reckoning 15. d. for an hour, or 3. d. 45. m. for a quarter of an hour, I finde three hours, three quarters (the diſtance of a quarter before four from midnight) to anſwer 56. d. 15. min. which being ſubtracted from 76. d. 30. min. the inclination of Meridians there remains

Imcl.mer.	hou.	quart.
76-- 30.	12	00
20-- 15	3	...
16-- 30	4	00
12-- 45	4	.
09-- 00	4	..
05-- 15	4	...
01-- 30	5	00
02-- 15	5	.
06-- 00	5	..
09-- 45	5	...
13-- 30	6	00
17-- 15	6	.
21-- 00	6	..
24-- 45	6	...
28-- 30	7	00
32-- 15	7	.
36-- 00	7	..
39-- 45	7	...
43-- 30	8	00
47-- 15	8	.
51-- 00	8	..
54-- 45	8	...
58-- 30	9	00
62-- 15	9	.
66-- 00	9	..
69-- 45	9	...
73-- 30	10	00

20. d. 15. m. for the diſtance of one quarter before four from the ſubſtyle. Again, from 20. d. 15. min. ſubtract 3. d. 45. min. (the quantity of degrees for one quarter of an hour) and there remains 16. d. 30. min. for the diſtance of the next line from the ſubſtyle, which is the hour of four in the morning. Thus for every quarter of an hour continue ſubtracting 3. d. 45. min. until your reſidue or remain be leſs than 3. d. 45. min. and then firſt ſubtracting that reſidue out of 3. d. 45. min. This new reſidue gives the quantity of degrees for that line on the other ſide the ſubſtyle. Now when you are paſſed to the other ſide of the ſubſtyle, continue adding 3. d. 45. min. to this laſt remain for every quarter of an hour, and ſo make up the table for what hours are proper to the plain.

3. Draw the ſquare ABCD. of what quantity the plain will admit, and make the angle CAG equal to the angle of the ſubſtyle with twelve. Again, croſs the line AG. in any two convenient points, as E. and F. at right angles by the lines KL. and CM.

4. Take the diſtance from the center unto 45. the radius to the leſſer lines of tangents, which is continued to 76. on the Sector ſide, enter this diſtance in 45. on the larger lines of tangents, and keeping the Sector at tha

gage, take out the tang. 20. d. 15. min. (which is the diſtance of the firſt line from the ſubſtyle) ſet this from 73. d. 30. min. (the diſtance of your laſt hour line from the ſubſtyle, as you ſee by the Table) toward the end upon the leſſer line of tangents, and where it toucheth as here at 75. 05. call that the gage tangent.

5. Enter the whole line KL. in the gage tangent, which in this example is 75. d. 05. min. and keeping the Sector at that gage, take out the tangent 73. 30. min. which is your laſt hour, and ſet from L. on the line KL. unto V. Again, take out the tangent 20. d. 15. min. which is your firſt line, and ſet it from K. towards V. and if it meet in V. it proves the truth of your work, and a line drawn through V. paralel unto AG. is the true ſubſtyle line. Then keeping the Sector at its former gage, ſet off the tangents of the hours, and quarters (as you finde them in the Table) from V. towards K. and from V. towards L. making points for them in the line KL. Laſtly, enter the radius of your tangents to theſe hour points in the radius of ſecants, and ſet off the ſecant of the ſtyles height from V. to T. Thus have you the hour points and ſtyle on one line of contingence. To mark them out upon the other line do thus.

Set the radius to the hour points upon the former line of contingence, from h. to p. on the line ChM. and entring hV. as Radius in the line of tangents, take out the tangent of the ſtyles height, and ſet from p. to r. Again, enter hr. radius in your line of tangents, and keeping the Sector at that gage, take out the tangents for each hour, and quarter, according to the table, and lay them down from h. to the proper ſide of the ſubſtyle toward C. or M. and applying a ruler to the reſpective points on KL. and CM. draw the lines for the hours and quarters. Laſtly, enter hr. radius on the lines of ſecants, and taking out the ſecant of the ſtyles height, ſet it from h. to S. and draw the line ST. for the ſtyle.

Sect. 6.

To deſcribe a direct polar Dial. See Fig. 15.

Draw BCDE. a rectangle paralellogram, from the middle of BC. to the middle of DE. draw the line 12. or ſubſtyle, appoint what place you pleaſe in BC. or CD. for the hour point of 7. in the morning, and 5. afternoon. Then,

Enter 12. 7.--12. 5. in the tangent 75. and ſet off 12. 1.--12. 11.--tang. 15. and 12. 2.--12. 10.--tang. 30. and 12. 3.--12. 9.--tang. 45.

45. Laſtly, 12. 4.--12. 8.--tang. 60. From theſe points draw the hour lines of 7. 8. 9. 10. 11. 12. 1. 2. 3. 4. 5. which are all the hours proper for theſe plains.

Sect. 7.

To draw a declining Polar. See Fig. 16.

1. By Chap. 2. Sect. 6. Par. 4. finde the inclination of Meridians, and diſtance of the ſubſtyle from the horizontal.

2. By Chap. 3. Sect. 5. Par. 2. make a Table for the diſtance of the hour points from the ſubſtyle.

3. Draw the ſquare BCDE. Set off the angle CAG. for the ſubſtyle, and croſs that ſubſtyle line at right angles in any two convenient places, as at H. and K. with the lines PHS. and RKT. for contingent lines.

4. Take any convenient length for your ſtyles height, and enter it radius in your line of tangents, keeping the Sector at that gage, prick off the hours from the ſubſtyle (by your table) upon both the contingent lines. Draw lines by the points in both contingents, and you have the hours:

For all other declining $\left.\begin{matrix}\text{re}\\\text{in}\end{matrix}\right\}$clining plains, it would be needleſs (I preſume) to inſiſt upon the deſcription of them: Sith ſo much hath been already mentioned, Chap. 2. S. 6. that there can ſcarcely be any miſtake, unleſs through meer wilfulneſs, or grand negligence.

Chap. IV.

To determine what hour lines are proper for any plain Superficies. By projection of the Sphere. See Fig. 17.

DRaw the Circle NESW. repreſenting the Horizon, and croſſing it into quadrants N. is North. S. South, E. Eaſt, W. Weſt, NS. the Meridian (which let be infinitely produced) Z. the center repreſents the Zenith. To finde the pole ſet half the co-tangent of the latitude from Z. toward N. it gives the point P. for the pole or the point through which all the hour lines muſt paſs.

The Suns declination in Cancer ſubtracted from the latitude, and the tangent of half the remain ſet from Z. to Π. gives the interſection of Cancer with the Meridian. Again, adde the complement of the Suns delineation in Cancer, unto the complement of your latitude, and the tangent of half that ſum ſet from Z. to ♆, gives the diameter of that tropick, half Π ♆, is the radius to deſcribe it. Half the tangent of your latitude ſet from Z.

to Q. gives that point for the interſection of the equator with the Meridian; and the co-ſecant of the latitude ſet from œ. toward N. gives the point ζ. the center of the equator.

Adde the Suns greateſt declination (or his declination in Capricorn) to the latitude, and the tangent of half that ſum ſet from Z. toward S. gives the point ♑, where Capricorn interſects the Meridian. Subtract the Suns declination in Capricorn from your latitude, and that remain ſubtract from 180. the tangent of half this laſt remain, ſet from Z. toward N. gives the point T. the diameter of Capricorn, and half the diſtance T ♑. is the radius to deſcribe it.

Set the ſecant of the latitude from P. towards S. it gives the point H. the center of the hour line of ſix croſs the line ZSH. at right angles in the point H. Then entring PH. Radius on the lines of tangents, ſet off the hour centers both wayes from H. reckoning 15. d. for an hour. Laſtly, ſetting one point of your Compaſſes in theſe center points, extend the other to P. and with that radius deſcribe the hour lines. Thus have you the ſphere projected, the following Sections will determine the hours for all plains.

Sect. 1.

To determine the hour lines for erect direct plains. *Fig.* 17.

The line N S. repreſents an erect Eaſt, and Weſt plain. That ſide next W. is Weſt, the other ſide next E. is Eaſt, where you may ſee that the Sun ſhines upon the Eaſt until twelve, or noon, and at that time comes upon the Weſt. The line WE. repreſents a direct North and South plain, the ſide next N. is North, the other next S. is South, where the North cuts the tropick of Cancer (which in the hour lines you finde 'twixt 7. and 8. in the morning; and again 'twixt 4. and 5. afternoon) is the time of the Suns going off, and coming on that plain. Where the South cuts the equator, which is in the points of ſix, and ſix is the time of the Suns going off, and coming on that plain.

Sect. 2.

To determin the hour lines for direct $\left.\begin{matrix}re\\in\end{matrix}\right\}$*clining Plains.* *Fig.* 17.

NBS. on the convex ſide is a Weſt incliner, where it cuts Capricorn, is the time of the Suns coming on that plain, afternoon. On the concave ſide its an Eaſt recliner, where it cuts Cancer, is the time of the Suns going off that plain, afternoon.

NCS. On the convex ſide is an Eaſt incliner, where it cus Capricorn, is the time of the Suns going off in the morning. On the concave ſide it is a Weſt recliner, where it cuts Cancer, is the time of the Suns coming on in the morning.

WDE. On the convex ſide is a South incliner, where until D. reach below œ. it hath all hours from ſix to ſix, and until D. reacheth below φ. it may have the twelve a clock line. But when D. reacheth below œ. draw a paralel of declination to paſs through the point D. and the interſection of that paralel with the limb of the circle NE SW. doth among the hour lines, ſhew the time of theSuns coming upon that plain in the morning,and going off again afternoon, when D. reacheth below φ. the interſection of the ark WDE. with the tropick of Capricorn, ſhews the time of the Suns going off that plain before noon, and coming on again, afternoon. And the interſection of the tropick of Capricorn with the limb ſhews the firſt hour in the morning the Sun comes on, and the laſt hour afternoon, that it ſtayes upon that plain. The convex ſide of WDE. is a north recliner, where it cuts Cancer, is the time of the Suns going off in the morning, and coming on again afternoon.

WFE. On the convex ſide is a North incliner, where it cuts Cancer, is the time of the Suns going off in the morning, and coming on afternoon. On the concave ſide is a South recliner, where until F. reach beyond P. it enjoyes the Sun only from ſix to ſix. When F. reacheth beyond P. where the ark cuts Cancer, you finde how much before ſix in the morning the Sun comes on, or after ſix at evening it goes off.

To draw any of theſe arks, Ex. gr. the ark NBS. do thus. Set the tangent of half the $\left.\begin{matrix}\text{re}\\\text{in}\end{matrix}\right\}$clination from Z. on the lire ZW. and it gives the point B. produce ZE. and ſet the co-ſecant of the $\left.\begin{matrix}\text{re}\\\text{in}\end{matrix}\right\}$clination from B. towards E. which reacheth to G. then G. is the center. & GB. the radius to draw that ark.

Note. The Semidiameter of the circle SENW. is radius to all the tangents, and ſecants, which you make uſe of for placing any oblique plain upon the Scheme.

Sect. 3.

To determin the hour lines for erect declining plains. Fig. 18.

For South-Eaſt, or North-Weſt plains.

By a line of chords ſet the angle of declination on the limb from W. toward S. as H. lay a ruler to HZ. and draw HZK, which on that ſide next NW. repreſents the North-Weſt, and where the line cuts Cancer, you have the time of the Suns coming on afternoon, and ſtaying until Sun ſet. But if it cut Cancer twice, then in the morning hours it ſhews what time the Sun goes off this plain, having all hours from Sun riſe to that time, and in the evening hours you have the time of the Suns coming on again, and ſtaying till ſun ſet. That ſide HZK. next SE. repreſents a South-Eaſt. Where the line cuts the equator in the evening hours, is the time of the Suns going off, where it cuts Cancer, in the morning hours, is the time of the Suns coming on that plain. For North-Eaſt, or South-Weſt, ſet the declination by a line of chords from E. towards S. as L. lay a ruler to ZL. and draw the line LZR, which on that ſide next NE. is North-Eaſt, where it cuts Cancer, is the time of the Suns going off in the morning. If it cuts Cancer twice, you have in the evening hours the time of the Suns coming on again, and ſtaying until Sun-ſet.

On that ſide next SW. is the South-weſt. Where it cuts the equator, or Capricorn in the morning hours, is the time of the Suns coming on, on; where it cuts Cancer in the evening hours, is the time of the Suns going off.

Sect. 4.

To determin the hours of declining $\left.\begin{matrix}\text{re}\\\text{in}\end{matrix}\right\}$ *clining Plains. Fig.* 18.

Firſt, ſet in the plain according to its declination. By Sect. 3. Ex. gr. LZR. a North Eaſt, or South-Weſt declining 50.d. 00.min. This done Croſs the line LZR. repreſenting the declination of the plain, at right angles in the point Z. as CZBG. Then for North-Eaſt incliners, or South-Weſt recliners, ſet half the tangent of the $\left.\begin{matrix}\text{re}\\\text{in}\end{matrix}\right\}$ clination from Z. toward C. as T. and ſet the co-ſecant of the $\left.\begin{matrix}\text{re}\\\text{in}\end{matrix}\right\}$ clination from T. toward B. as TG. Then G. is the center, and GT. the radius to deſcribe the ark RTL. Whoſe convex ſide repreſents a North-Eaſt incliner, where it cuts Libra or Capricorn, is the time of the Suns going off in the morning; if it cuts Cancer twice, the interſection of Cancer with the evening hours ſhews what time the Sun comes again upon ſuch a plain afternoon, and continues till Sun ſetting. The concave ſide is a South-Weſt recliner, where it cuts

Cancer in the morning hours, is the time of the Suns coming on, in caſe it interſects Cancer twice in the evening hours, you have the time that the Sun goes off.

For a North-Eaſt recliner, or South-Weſt incliner, ſet the point T. from Z. toward B. and the point G. from T. (ſo placed) toward C. and draw the ark on that ſide RZL. toward B. whoſe convex ſide will repreſent a South-Weſt incliner, and where the ark cuts the equator or Capricorn, you have the time of the Suns coming on that plain. The concave ſide is a North-Eaſt recliner, where the ark cuts Cancer, is the time for the Suns going off that plain. When the ark cuts Cancer twice, the Sun comes on again before it ſets.

For a North-Weſt recliner, or South-Eaſt incliner. Enter the declination by Sect. 3. as HZK. Croſs it in the point Z. at right angles, as OZD. ſet half the tangent of the $\left.\begin{matrix}\text{re}\\ \text{in}\end{matrix}\right\}$clination from Z. toward O. as V. and the coſecant of the $\left.\begin{matrix}\text{re}\\ \text{in}\end{matrix}\right\}$clination from V. toward D. as F. then is F. the center, and FV. the radius to draw the ark HVK. Where it cuts Cancer the hour lines, tell you the time of the Suns going off in the morning, and entring again afternoon, upon the North-Weſt recliner. Where it cuts the equator you have the time of the Suns going off the South-Eaſt, where it cuts Cancer in the morning hours is the time of the Suns coming on that plain.

For a North-Weſt incliner, or South-Eaſt recliner, ſet the point V. from Z. toward D. and the point F. ſet from V. (ſo placed) toward O. and draw the ark on that ſide Z. next D. Then where the convex ſide cuts Cancer, you have the time of the Suns going off in the morning, and coming on again afternoon upon the North-Weſt incliner. Where the concave ſide cuts the equator, you have the time of the Suns going off the South-Eaſt recliner; where it interſects Cancer, is the time of his coming on that plain in the morning.

Note. All the precedent rules about plains are appropriated to us that live in Northern Hemiſphere, In caſe any one would apply them to the South Hemiſphere: What is here called North, there name South, and what we here term South, there call North, and the rules are the ſame.

——Si quid noviſti { *plenius* / *promptius* / *rectius* } *iſtis,*
Candidus imperti: Si non, His utere mecum.

FINIS.

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